Reconstruction of a Three-Dimensional Tableau from a Single Realist Painting

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***Abstract*—We reconstructed a three-dimensional tableau from a single realist painting–Scott Fraser's "Three way vanitas" (2006)–based on multiple stereo reconstruction applied to the direct image and the images in three plane mirrors depicted within the painting. The tableau contains a carefully chosen complex arrangement of objects including a moth, egg cup, and strand of string, glass of water, bone, and hand mirror. Each of the three plane mirrors presents a different view of the tableau from a virtual camera behind each mirror and symmetric to the artist's viewing point. Our new contributions are three folds. First, we incorporate single-view geometric information extracted from the direct image of the wooden mirror frames in obtaining the camera models of these three virtual cameras. Second, we estimate 3D of objects using multiple stereo pairs within a single painting. Third, the geometric accuracies of the painting are also evaluated.**

I. Introduction

The problem of reconstructing a three-dimensional scene from multiple views is well explored, and a number of general methods, such as those based on correlation, relaxation, dynamic programming, have been developed and fully characterized [1], [2]. Three-dimensional reconstruction and metrology can be based on single views as well [3], [4], and recently Criminisi and his colleagues have applied such techniques to the analysis of paintings, for instance reconstructing the virtual spaces in Masaccio's Holy Trinity (c. 1425), Piero della Francesca's Flagellation of Christ (c. 1453), Hendrick V. Steenwick's St. Jerome in his study (1630), Jan Vermeer's A lady at the virginals with a gentleman (16621665), and others [5]. These methods reveal both the high geometric accuracies in some passages, and the geometric inconsistencies in others, properties that are nearly impossible to determine by eye. Such analyses shed new light on these works and the artists' working methods, for instance revealing whether an artist likely used geometrical aids during the execution of their work.

Recently Smith, Stork and Zhang reconstructed the three-dimensional space depicted in a highly realistic modern painting, Scott Fraser's Three way vanitas (Fig. 1) using traditional multiple-view reconstruction methods applied to the direct view and a view visible in a depicted mirror [6]. They found passages within the painting having good spatial agreement, showing high accuracy of the painter, but also passages of significant disagreement, such as the height of the water in the glass in the direct view and the view in the right-hand mirror.

Even though using reflected images by mirrors is a very popular approach for stereo vision in computer vision [7]–[10], it was the first time to analyze a painting with such a setup. There were some limitations in that previous work as well as unexplored opportunities. For instance, each reconstruction was based on just the direct view and a single reflected view. In fact, though, several volumes within the tableau are visible in three or more views and could have been reconstructed using all those views. Furthermore, the images of the frames of the mirrors provide geometric constraints about the centers of projection of the images depicted within each mirror, and the earlier scholarship did not incorporate that information when reconstructing the three-dimensional space. It is this latter opportunity we address here. However, 3D reconstruction is not a goal in itself, but merely to further evaluate the painting accuracy estimation.

The paper is organized as the following. Section II describes the painting, previous scholarship and an overview of our new approach to three-dimensional reconstruction based on incorporating single-view geometrical constraints into multi-view image correspondences. Section III briefly describe the single-view geometric estimation of camera parameters. Section IV verifies the accuracy of the painting based on multi-view epipolar geometry. Section V further verifies the accuracy of the painting based on multi-view three-dimensional reconstruction. Section VI summarizes our conclusions and describes future directions.

II. The work and problem addressed

Fig. 1 shows the work we consider, Scott Fraser's Three way vanitas (2006). This painting was commissioned as part of The Object Project [11], in which fifteen artists were commissioned to create works, each containing five specified objects: hand

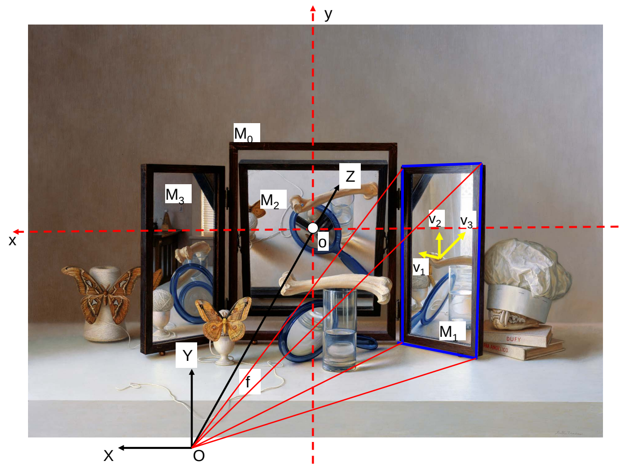


Fig. 1. The work, the notations and the constraints.

mirror, bone, moth, ball of string and drinking glass [12]. Fraser's contribution is a sober meditation in the vanitas genre reminding the viewer of the transient nature of fame and vanity and of the inevitability of death executed in a narrow palette of white, brown and blue.

The problem we address is how to use geometric information of the frames of the mirrors to obtain an estimate of the centers of projections of the images depicted in each mirror, and how to integrate this information into estimates based on the images themselves so as to obtain an improved overall estimate of the camera models and hence evaluation of the three-dimensional scene itself.

Then three pairs of stereoscopic images can be formed. Here the pair of images were provided by the direct view from the artist's viewpoint (main camera) and the image depicted in each mirror (virtual cameras). We used the fact that the images depicted in the mirrors corresponded to the views from the positions of the virtual cameras, but also that these depicted images were themselves viewed from the artist's viewpoint. Our resulting three-dimensional reconstruction yielded more robust 3D reconstruction than was obtained in previous multi-view approaches [6], which used only limited number of points on the table in both the real image and the mirror images but not the single-view geometrical estimate.

III. Camera Parameter Estimation

This paper will focus on how to use the camera and mirror geometry to infer 3D structure of the painting and to analyze the accuracy of the drawing. For completion, we provide a brief summary of our calibration method proposed in [13].

We label the mirrors, reading right to left, M_i, their associated reflected images I_i, and corresponding centers of projection C_i, for $i = 1, 2, 3$. The middle frame is labeled as M_0.

In [13], we have described details in estimating the following parameters of the real camera and the three virtual cameras created by the three mirrors: (1) the image center $o(c_x, c_y)$; (2) the focal length f; (3) the plane representation of each mirror; and (4) the pose (location and orientation) of the three virtual cameras related to the real camera (the artists eye). Here we provide a brief summary. The four cameras share the same focal length and the image center, but the images of the three mirrored cameras are flipped in the x direction. The intrinsic geometric constraints we use are the following:

1) All three mirrors are rectangular.
2) The two flanking mirrors are the same width.
3) The back edges of the two flanking mirrors are at the same distance.
4) The aspect ratio of the image is 1:1.

In addition to the above constraints, by analyzing the images of the frames and mirrors, we have also observed that the left and right flanking mirrors (M_3 and M_1) and the central frame (M_0) are vertical, and the middle inset mirror (M_1) is only tilted in the y direction. We will use the image height (H) and width (W) of the frame M_0 in the middle as the reference size of all the real objects (including mirrors). As measured in the image, we have $H = 643.2$ pixels, and $W = 543.0$ pixels.

To find the image center, we need to have both a set of 3D horizontal lines and a set of vertical lines whose projections are not parallel in the image. We used the mirror frames for the task and the image center is estimated as $(c_x, c_y) = (868.6, 526.8)$. From this point on, we will use the image coordinate system xoy (Fig. 1), by performing a transformation of $x = c_x - x_i, y = c_y - y_i$, where (x_i, y_i) is represented in the original digital image system with the origin at the top-left corner.

We found the focal length f by using the fact that the left and right flanking mirrors have exactly the same width. We obtain $f = 2050.7$ (pixels).

Once we obtain the value of the focal length f and the image center, we can calculate the 3D vectors for the directions of the two edges of each mirror M_i using their vanishing points, and then its normal by using the cross product of the two vectors. In order to find the virtual camera parameters mirrored by each mirror, we will build a world coordinate system on each mirror. For example, for mirror M_1, after we find the three vectors and then normalized them into column unit vectors, represented as v_1, v_2, v_3, we can define a world coordinate system using the middle of the mirror plane as its origin, and the three vectors as its three coordinate axes. In general, the transformation between the camera coordinate system and the world coordinate system of the mirror $M_i (i = 1, 2, 3)$ can be represented as

$$P_c = R_{1i} P_w + T_{1i} \tag{1}$$

where $P_c = (X_c, Y_c, Z_c)^t$ is represented in the camera coordinate system $OXYZ$, $P_w = (X_w, Y_w, Z_w)^t$ is represented in the mirror coordinate system, $R_{1i} = (r_{pq}) 3 \times 3$, which is (v_1, v_2, v_3) for mirror M_1, and T_{1i} to be determined. The projection of P_c into the image of the main camera is

$$(x, y) = (f \frac{X_c}{Z_c}, f \frac{Y_c}{Z_c}) \tag{2}$$

To find the translational vectors for all the three mirrors, we use the dimension of the middle frame M_0 as a reference.

Then the mirrored coordinate system, *i.e.,* the virtual camera C_i, can be easily obtained. Here we use a coordinate transformation method to find the relation between each virtual camera C_i and the real camera C, by finding the rotation matrix R_i and translational vector T_i $(i = 1, 2, 3)$. In our implementation, we use (1) to represent the origin and the three axes of the camera coordinate system in the world coordinate system $X_w Y_w Z_w$ of each mirror M_i. Since $X_w O Y_w$ is the mirror plane, the mirrored origin and axes of the main camera can be simply obtained by changing the signs of their Z_w components. Then we do a similar procedure as in (1) to find the transformation (characterized by R_{2i} and T_{2i}, $i = 1, 2, 3$) between the world coordinate system M_i and the virtual camera C_i:

$$P_w = R_{2i} P_i + T_{2i} \tag{3}$$

Combining (1) and (3), we can find the transformation between the real camera and the virtual camera:

$$P_c = R_i P_i + T_i \tag{4}$$

$$R_i = R_{1i} R_{2i}, T_i = R_{1i} T_{2i} + T_{1i} \tag{5}$$

Table I shows the estimated results. Note that for consistency, we used right-handed coordinate systems for all the three virtual cameras. But in reality, the mirrored virtual cameras should all have left-handed systems, as noted in [7]. Therefore in all the three virtual views (in the mirrors), we will have to change the signs of the x coordinates in all calculations.

The calibration results here were also verified by using the method as noted in [7]. This will be further verified using epipolar geometry of stereo vision.

IV. Painting Verification: Epipolar Geometry

A set of representative corresponding points are manually selected in the four views (the main camera view and the three mirror views) for verifying the accuracy of the painting and the calibration, and for reconstructing the 3D structure of those objects. Fig. 2 shows the selected points in the main camera view marked with numbers from 0 to 19.

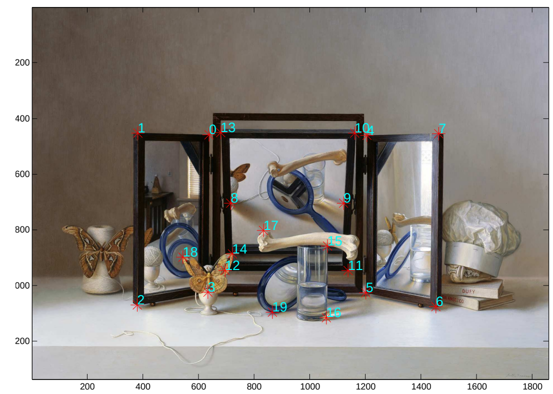


Fig. 2. Selected points are used for epipolar geometry and 3D reconstruction.

A. Epipolar geometry

Let $P_c(X_c, Y_c, Z_c)$ and $P_i(X_i, Y_i, Z_i)$ be vectors of the same 3D point P in the real camera coordinate system and the ith virtual camera coordinate system $(i = 1, 2, 3)$, respectively, as in (4). From (4), the following relationship holds:

$$P_i = R_i^T (P_c - T_i) \tag{6}$$

We calculate the essential matrix E_i $(i = 1, 2, 3)$ between each virtual camera C_i and the real camera C:

$$E_i = R_i^T S_i \tag{7}$$

where S_i is a 3×3 matrix generated from T_i. Then for a pair of points in the two views, virtual view p_i and real view p, we have

$$p_i^T E p = 0 \tag{8}$$

where p_i = $(-x_i, y_i, f)^T$ is a point in the i^{th} virtual camera view, and $p = (x, y, f)^T$ is a point in the main camera view, of the same 3D point P. Note that we put a sign in front of the x_i coordinate of the mirrored point of p, which should be (x_i, y_i, f). The pair of points, (x_i, y_i, f) and (x, y, f), actually shares the same image coordinate system xoy, as in Fig. 1.

Given p, (8) defines an epipolar line in the i^{th} virtual view. The corresponding point p_i should be on the line. By defining $a = Ep = (a_1, a_1, a_3)^T$, which is a column vector, we have the epipolar line equation

$$(-x_i, y_i, f)(a_1, a_1, a_3)^T = 0 \tag{9}$$

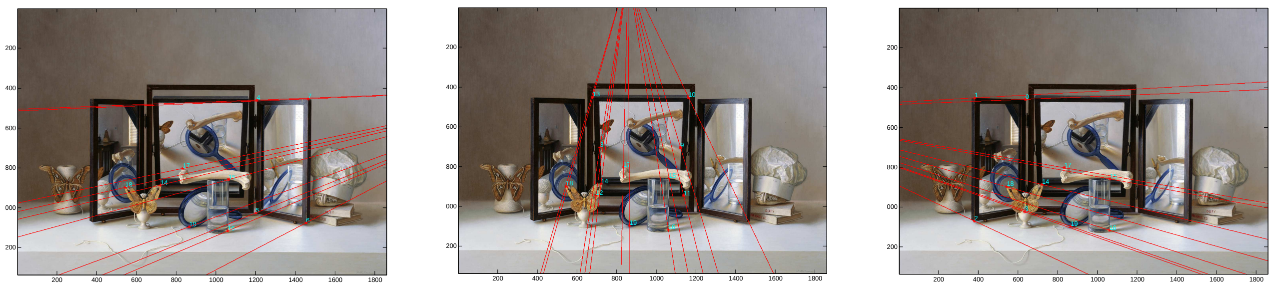
and the"mirrored" epipolar line of the point p that can be drawn on the "shared" image space as

$$(x_i, y_i, f)(-a_1, a_1, a_3)^T = 0 \tag{10}$$

Fig. 3 shows the selected points in the main camera view marked with numbers from 0 to 19, and their corresponding "mirrored" epipolar lines in three mirrored views, respectively.

TABLE I
THE TRANSFORMATIONS BETWEEN VIRTUAL CAMERA C_i ($i = 1, 2, 3$) AND THE MAIN CAMERA C

M_i	M_1	M_2	M_3
R_i	$\begin{matrix} -0.0846 & 0.0610 & 0.9945 \\ -0.0610 & 0.9959 & -0.0663 \\ -0.9945 & -0.0663 & -0.0806 \end{matrix}$	$\begin{matrix} -0.9999 & -0.0035 & -0.0103 \\ 0.0035 & 0.7878 & -0.6159 \\ 0.0103 & -0.6159 & -0.7877 \end{matrix}$	$\begin{matrix} -0.1992 & -0.0049 & -0.9800 \\ 0.0049 & 1.0000 & -0.0059 \\ 0.9800 & -0.0059 & -0.1991 \end{matrix}$
T_i(measured in pixel)	$(-2299.7, 153.4, 2498.7)$	$(20.4, 1224.4, 3553.8)$	$(2200.2, 13.3, 2692.3)$



(a) (b) (c)

Fig. 3. Manually picked points in the main camera view and their "mirrored" epipolar lines in the (a) right (b) middle (c) left mirror view

B. Discussions: epipolar lines and painting accuracy

We have several observations of the epipolar geometry, and the accuracy of the drawing.

1) All the epipolar lines intersect at a common point, the epipole, which is the projection of the center of the real camera in the virtual camera view. This can be seen from the three sets of epipolar lines in three virtual views.
2) For any point p in the real camera view, its "mirrored" epipolar line also passes through this point. This is an interesting property of the stereo system constructed by using a mirror.

   Here is a simple proof for (2):

   a) If the 3D point P is on the mirror, then its "mirrored" point is the same as the real point, therefore $p = (x_i, y_i, f)$. Hence p is on the line represented in (10).
   b) If the 3D point P is in front of the mirror, then extend O_cP so that it intersects with the mirror at P_m, the projection of P_m in the real camera view is p_m, which is exactly the same as p. From (2a) we know p_m is on the line represented in (10). Therefore p is on the same mirrored epipolar line.
3) There seem to have system errors in the painting judged by the epipolar geometry. The corresponding points in the right mirrored views are obviously all below their epipolar lines, and those in the left mirrored views are all above their epipolar lines. Given the assumptions we made on the mirrors and the camera, the mirrored points of the objects in the right mirror seem to be too low, and on the other hands, in the left mirror seem to be too high. This can be verified by comparing the heights of the object points in the left and right mirrors. This seems to indicate the drawings in the mirrors are not perspectively accurate. We have also noticed that the perspective distortions of the depth edges of the left and right mirrors are more than they should be. All these problems may be due to the geometric constraints made in Section III, but these constraints were confirmed by the painter.
4) The epipolar geometry in the middle mirror seems to be the most accurate. This is also verified in the following 3D estimate step.

V. PAINTING VERIFICATION: 3D RECONSTRUCTION

A. 3D reconstruction

From each pair $\{p_i, p\}$ we can get $P_c^{(i)}(X, Y, Z)$, represented in the real camera coordinate system ($i = 1, 2, 3$). From the four views, we get three estimates, the average can be calculated. We found that even though the drawing does not pass the accurate epipolar geometry test, the perspective effects are correct within certain error bounds and we can still obtain 3D estimations for points both on mirrors and off mirrors.

This reconstruction is solved by stereo triangulation [14] since we already know both the extrinsic and intrinsic parameters. Because of error in the locations of corresponding points, the two rays, O_ip_i and Op, will not intersect exactly in space and thus we have to approximate it by finding the closest midpoint between these two lines.

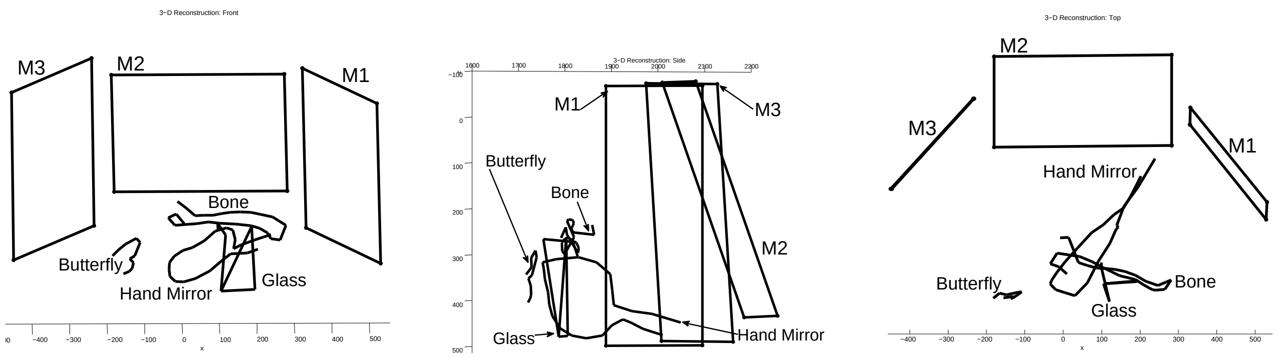
Note that for the points on mirror frames, only a pair of views can be used, and the images points in the real camera view and mirrored view are exactly the same. The points on regular objects are reconstructed using as many pairs (1 to 3) as possible.

Table II shows the reconstructed points using multiple stereo views. If a point is visible from more than one view, we reconstructed the point for each view, find its average and the average error in 2 or 3 estimations. All the values are

TABLE II
MULTI-VIEW 3D RECONSTRUCTION IN PIXEL*

Pts	M_1 view (right)	M_2 view (middle)	M_3 view (left)	Average	Average error (RMS)
14	-	(109.2, -295.9, 1680.4)	(125.9, -303.0, 1790.2)	(117.6, -299.5, 1735.3)	(8.4, 3.6, 54.9)
15	-	(-174.0, -265.4, 1657.9)	-	(-174.0, -265.4, 1657.9)	-
16	(-170.8, -518.4, 1754.2)	(-191.6, -487.4, 1715.0)	-	(-181.2, -502.9, 1734.6)	(10.4, 15.5, 19.6)
17	(25.8, -266.4, 1822.4)	(15.8, -240.1, 17874.7)	(29.1, -236.9, 1842.1)	(23.6, -247.8, 1816.4)	(5.7, 13.2, 23.8)
18	(280.1, -336.8, 1785.0)	-	-	(280.1, -336.8, 1785.0)	-
19	(-3.4, -535.9, 1863.5)	(-15.0, -488.2, 1760.7)	(0.3, -513.6, 1885.6)	(-6.1, -512.6, 1836.6)	(6.5, 19.5, 54.4)

* Use Fig. 2 for points reference



(a) (b) (c)

Fig. 4. 3D reconstructed objects and mirrors in the (a) front (b) side (c) top view

measured in pixels. Average errors vary, from several pixels to more than fifty pixels.

Fig. 4 shows the 3D reconstruction of both objects and mirrors in the painting. The location of each object is relatively similar to the human perception of the painting. We further verify this in subsequent sections.

B. 3D reconstruction verification

1) Using special points: By picking the frames corners and compare them with "ground-truth" data, we want to verify the accuracy of the drawing and the 3D reconstruction technique using stereo.

For this, we picked 8 corners from the left and right frames, obtained its image coordinates and transform it into camera coordinates. Note that for each point on the surface of a mirror, its projection in the real camera and in the mirrored virtual camera shares the same image coordinates. Using the stereo reconstruction technique, we calculated its camera coordinate. For these points, the two rays of each pair of points intersect on its 3D location.

To verify the correctness, using (1), we computed the world coordinates $P_w = (P_c - T_{1i})R_{1i}^T$, where P_c is camera coordinate, T_{1i} is the corresponding frames translation, and R_{1i} is the corresponding frames rotation. The computed shape and dimensions of both mirrors are fairly accurate. The Z_w values, are very close to zeros, which are the ground truth values.

Another way to verify the accuracy of the calibration is to use a single view and compute some world coordinates. Again we are going to use the frames corners for verification. We can do this by solving (1) and (2) with $Z_w = 0$ because we are computing the coordinates of a point on the mirror plane. The computed results are almost identical to those using stereo vision method.

2) Using regular object points: Because of errors in the locations of corresponding points, probably mainly due to the drawing inconsistencies, the two rays, $O_i p_i$ and Op, will not intersect exactly in space. Hence, we have to approximate it by finding the closest midpoint between these two lines. Table III shows the corresponding points P_1 and P_2 on two rays that are closed to the midpoint for each pair of points, for regular object points, # 14-19 (except # 18 which cannot be seen in the middle mirror), using the main camera and the 2^{nd} virtual camera generated by the middle mirror M_2. The average differences between P_1 and P_2 in X, Y and Z directions are 31.0, 4.3 and 1.2 (pixels). Note this result is consistent with the epipolar geometry in that this pair is vertically aligned and the epipolar lines are mainly off in the X direction. Similar results are observed from the views of the left and right virtual cameras, where the errors in the Y direction are the largest due to the misalignment in that direction in the painting. The average error for the right and left virtual camera is (10.2, 32.4, 6.9) and (4.8, 20.1, 4.0), respectively.

VI. CONCLUSIONS AND DISCUSSIONS

We reconstructed a three-dimensional tableau from a single realist painting–Scott Fraser's "Three way vanitas" (2006)–

TABLE III
TRIANGULATION ERRORS IN PIXEL UNIT USING THE STEREO PAIR OF MAIN CAMERA AND MIDDLE VIRTUAL CAMERA*

	P_1			P_2			differences		
Pts	X_c	Y_c	Z_c	X_c	Y_c	Z_c	dX	dY	dZ
14	120.6	−294.6	1679.8	97.7	−297.2	1681.0	22.9	2.6	1.2
15	−158.9	−268.7	1658.8	−189.1	−262.1	1657.0	30.2	6.6	1.8
16	−161.0	−493.1	1716.2	−222.2	−481.7	1713.8	61.2	11.4	2.4
17	29.9	−240.0	1784.5	1.7	−240.1	1785.0	28.2	0.1	0.5
19	3.1	−488.6	1760.6	−33.2	−487.8	1760.9	36.3	0.8	0.3
Average error							35.76	4.2	1.2

* Use Fig. 2 for points reference

based on multiple stereo reconstruction applied to the direct image and the images in three plane mirrors depicted within the painting. Our method for estimating the camera models for the virtual cameras and the 3D structure of the objects was based on the single-image information of the mirror frames in the primary image of the painting, the image seen from the artist's viewing point (the main camera).

Here are some conclusions and observations.

1) single-view analysis: The relative 3D structures of the rectangles mirrors and frames are estimated by using their perspective analysis with a few assumptions of their geometry (*i.e.* rectangular shapes) that can be easily obtained.
2) camera calibration: Both the intrinsic and extrinsic parameters of the main camera and the virtual cameras created by mirrors are fully recovered.
3) 3D estimation: The 3D estimates of both the frames and regular objects are consistent among the single-view analysis, and results from multiple stereo triangulation. Comparing to the results published in [6], our 3D estimation is more accurate because we obtained more robust camera geometry by using structure of mirror frames rather than using a limited number of points concentrating in the direct view and the frontal mirror, and incorporated results from multiple views to reconstruct. They reported there are some clear inconsistencies, such as the bone and the water glass are not the same distance away. On the other hand, our reconstruction shows that the bone and the water glass are approximately the same distance away.
4) painting analysis: Overall, the painting has 3D geometric consistencies among the mirrors and among the objects. However, they are some stereo and perspective inconsistencies, across four views. These could either be the accuracies of the perspective distortions, orientations and sizes of the mirrors, or of the locations of objects inside the mirrors, which cannot be easily observed by eye.

In summary, overall method is applicable to improving the reconstruction of three-dimensional scenes based on general containing plane mirrors or even curved mirrors (after the mirror image is dewarped), such as in digital photographs and video. Our methods provide an objective test of the spatial accuracy of realist artists. Moreover, the three-dimensional tableaus reconstructed from paintings can shed light on the working methods of some realist painters by providing new views into the picture space and thus the artist's three-dimensional compositional choices. As such, our work extends the new discipline of computer vision applied to the study of fine art.

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