3D Vision

CSc I6716
Fall 2010

Topic 3 of Part II
Stereo Vision

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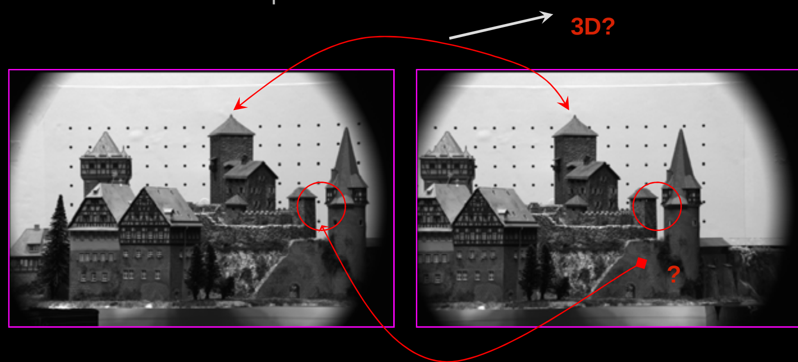
3D Computer Vision
and Video Computing

Stereo Vision

- Problem
  - Infer 3D structure of a scene from two or more images taken from different viewpoints
- Two primary Sub-problems
  - Correspondence problem (stereo match) -> disparity map
    - "Similar" instead of "Same"
    - Occlusion problem: some parts of the scene are visible only in one eye
  - Reconstruction problem -> 3D
    - What we need to know about the cameras' parameters
    - Often a stereo calibration problem
- Lectures on Stereo Vision
  - Stereo Geometry – Epipolar Geometry (*)
  - Correspondence Problem (*) – Two classes of approaches
  - 3D Reconstruction Problems – Three approaches

A Stereo Pair

- Problems
  - Correspondence problem (stereo match) -> disparity map
  - Reconstruction problem -> 3D



CMU CIL Stereo Dataset : Castle sequence
http://www-2.cs.cmu.edu/afs/cs/project/cil/ftp/html/cil-ster.html

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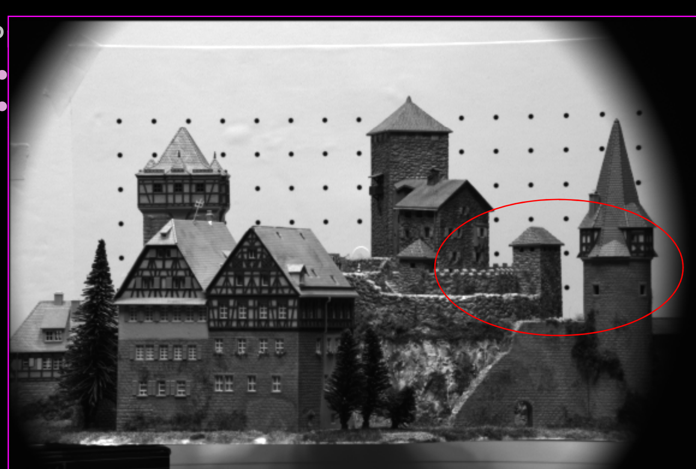
More Images…

- P



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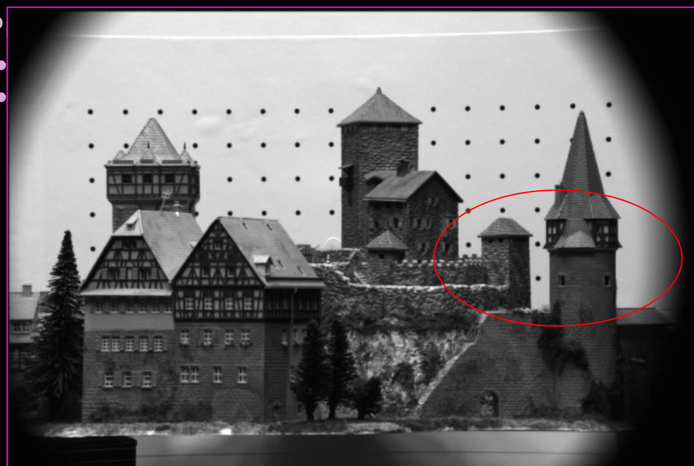

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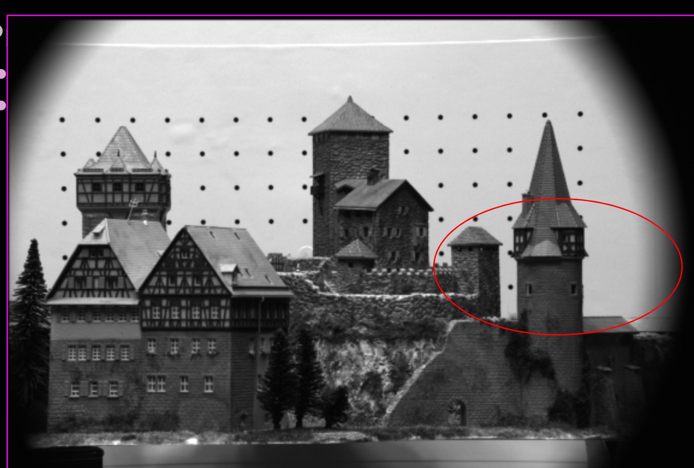

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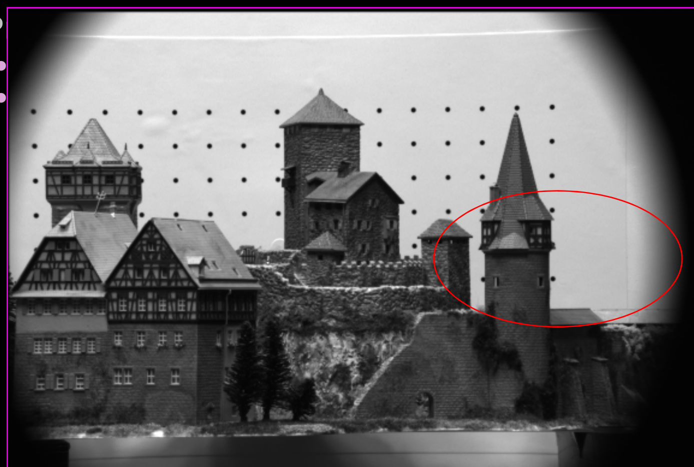

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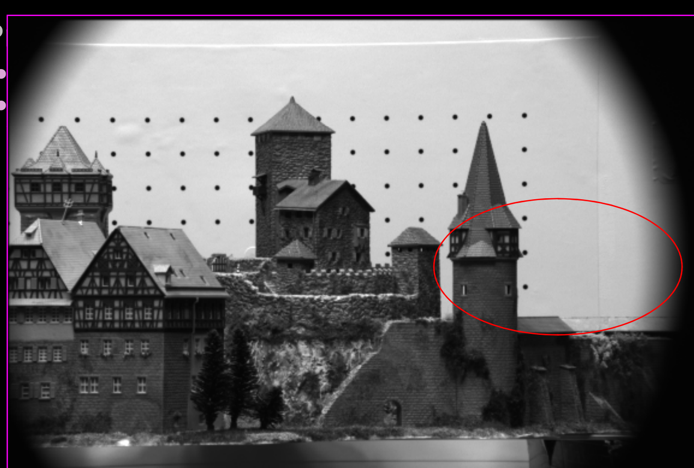

Part I. Stereo Geometry

- A Simple Stereo Vision System
  - Disparity Equation
  - Depth Resolution
  - Fixated Stereo System
    - Zero-disparity Horopter
- Epipolar Geometry
  - Epipolar lines – Where to search correspondences
    - Epipolar Plane, Epipolar Lines and Epipoles
    - http://www.ai.sri.com/~luong/research/Meta3DViewer/EpipolarGeo.html
  - Essential Matrix and Fundamental Matrix
    - Computing E & F by the Eight-Point Algorithm
    - Computing the Epipoles
- Stereo Rectification

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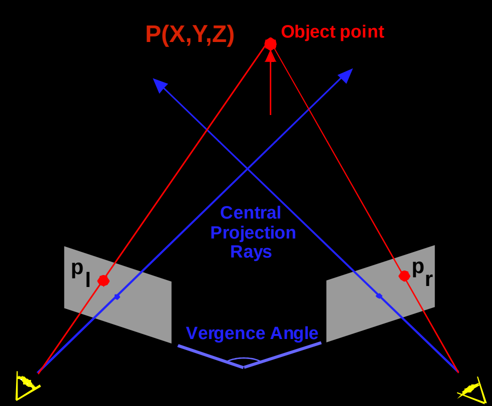
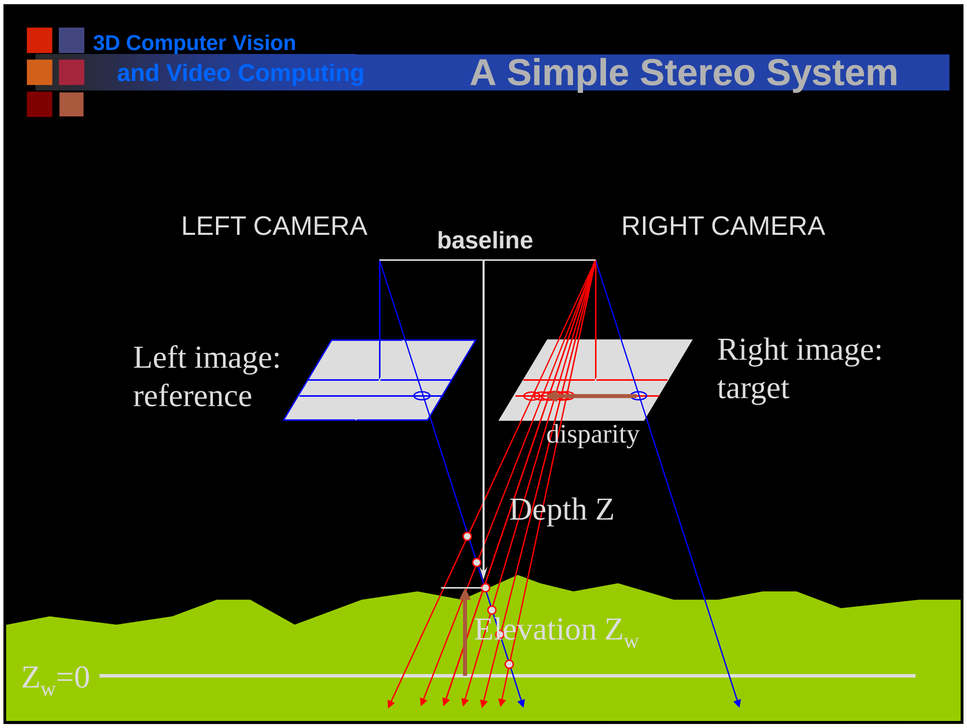
Stereo Geometry



- Converging Axes – Usual setup of human eyes
- Depth obtained by triangulation
- Correspondence problem: p_l and p_r correspond to the left and right projections of P, respectively.



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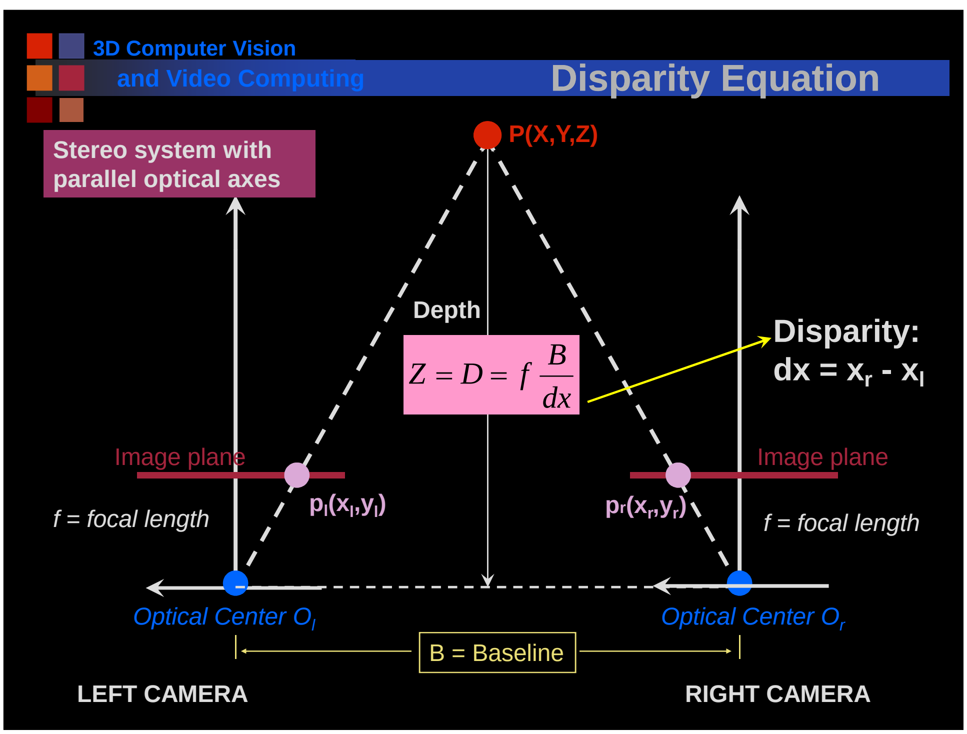
A Simple Stereo System
LEFT CAMERA
baseline
RIGHT CAMERA
Left image:
reference
Right image:
target
disparity
Depth Z
Elevation Z_w
$Z_w=0$



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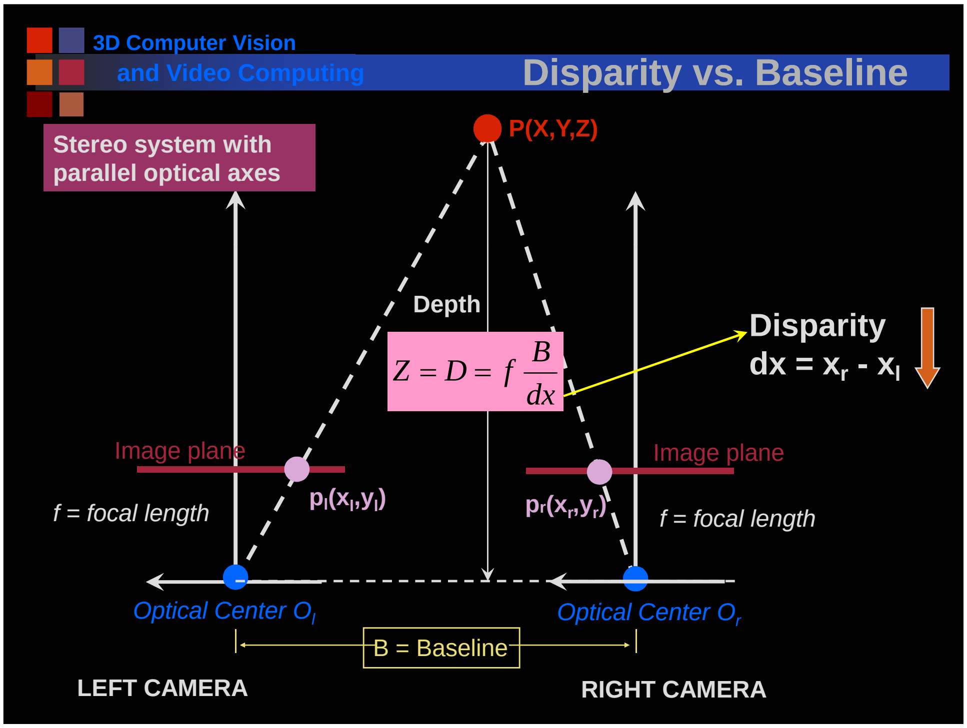
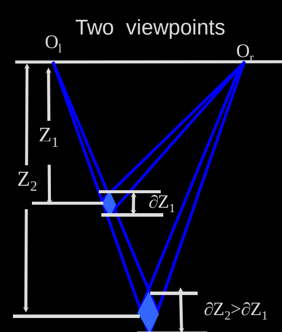
Disparity Equation
Stereo system with
parallel optical axes
P(X,Y,Z)
Depth
$Z = D = f \frac{B}{dx}$
Disparity:
$dx = x_r - x_l$
Image plane
Image plane
$p_l(x_l,y_l)$
$p_r(x_r,y_r)$
f = focal length
f = focal length
Optical Center O_l
Optical Center O_r
B = Baseline
LEFT CAMERA
RIGHT CAMERA



Depth Accuracy

- Given the same image localization error
  - Angle of cones in the figure
- Depth Accuracy (Depth Resolution) vs. Baseline
  - Depth Error $\propto$ 1/B (Baseline length)
  - PROS of Longer baseline,
    - better depth estimation
  - CONS
    - smaller common FOV
    - Correspondence harder due to occlusion
- Depth Accuracy (Depth Resolution) vs. Depth
  - Disparity (>0) $\propto$ 1/ Depth
  - Depth Error $\propto$ Depth^2
  - Nearer the point, better the depth estimation
- An Example
  - f = 16 x 512/8 pixels, B = 0.5 m
  - Depth error vs. depth



Absolute error

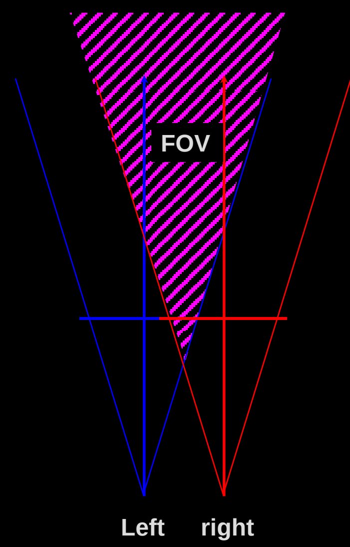
$$\partial Z = \frac{Z^2}{fB} \partial(dx)$$

Relative error

$$\frac{\partial Z}{Z} = \frac{Z}{fB} \partial(dx)$$

Stereo with Converging Cameras

- Stereo with Parallel Axes
  - Short baseline
    - large common FOV
    - large depth error
  - Long baseline
    - small depth error
    - small common FOV
    - More occlusion problems
- Two optical axes intersect at the Fixation Point
  - converging angle θ
  - The common FOV Increases



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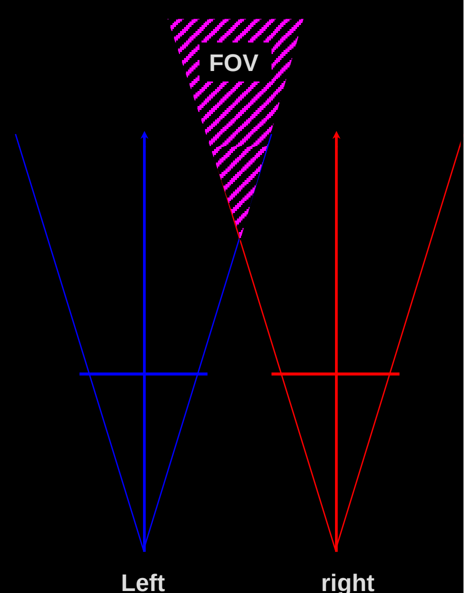

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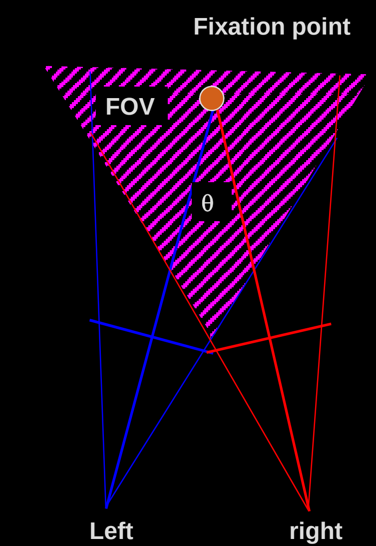
Stereo with Converging Cameras

- Two optical axes intersect at the Fixation Point
  - converging angle θ
  - The common FOV Increases
- Disparity properties
  - Disparity uses angle instead of distance
  - Zero disparity at fixation point
    - and the Zero-disparity horopter
  - Disparity increases with the distance of objects from the fixation points
    - >0 : outside of the horopter
    - <0 : inside the horopter
- Depth Accuracy vs. Depth
  - Depth Error $\propto$ Depth^2
  - Nearer the point, better the depth estimation



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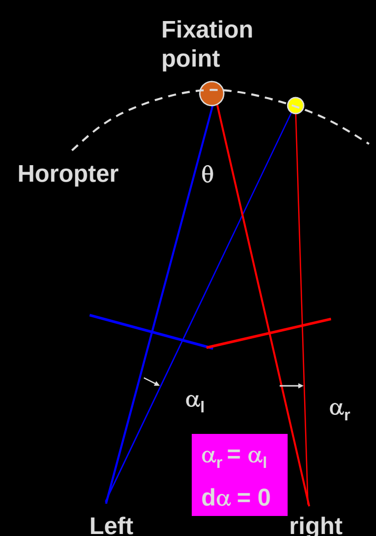

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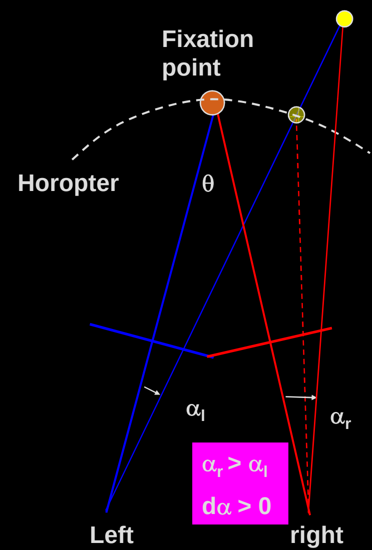

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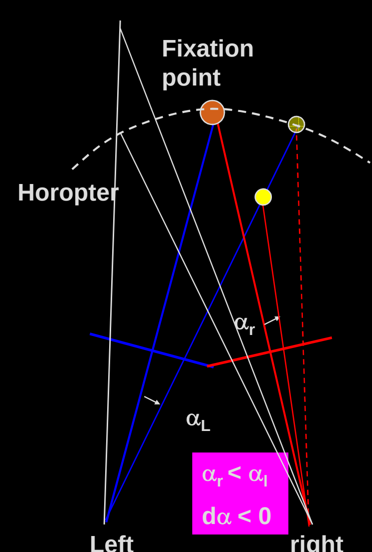

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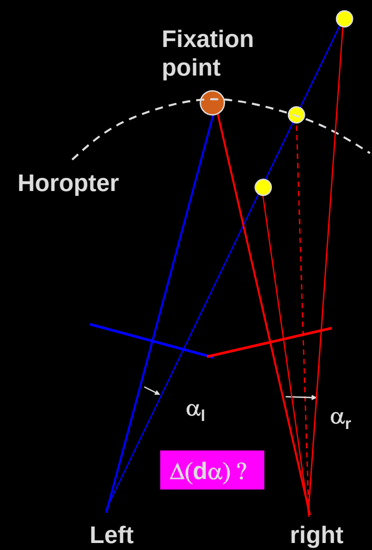
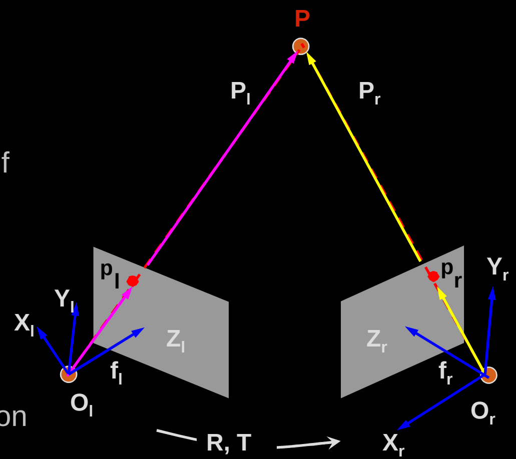


Break

- Homework #4 online, due on November 29 before class

Parameters of a Stereo System

- Intrinsic Parameters
  - Characterize the transformation from camera to pixel coordinate systems of each camera
  - Focal length, image center, aspect ratio
- Extrinsic parameters
  - Describe the relative position and orientation of the two cameras
  - Rotation matrix R and translation vector T

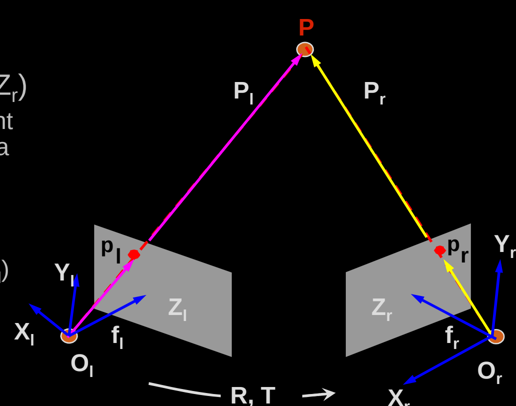


Epipolar Geometry

- Notations
  - $P_l = (X_l, Y_l, Z_l)$, $P_r = (X_r, Y_r, Z_r)$
    - Vectors of the same 3-D point P, in the left and right camera coordinate systems respectively
  - Extrinsic Parameters
    - Translation Vector $T = (O_r - O_l)$
    - Rotation Matrix R

$$\mathbf{P_r} = \mathbf{R}(\mathbf{P_l} - \mathbf{T})$$

  - $p_l = (x_l, y_l, z_l)$, $p_r = (x_r, y_r, z_r)$
    - Projections of P on the left and right image plane respectively
    - For all image points, we have $z_l = f_l$, $z_r = f_r$



$$\mathbf{p}_l = \frac{f_l}{Z_l}\mathbf{P_l} \qquad \mathbf{p}_r = \frac{f_r}{Z_r}\mathbf{P}_r$$

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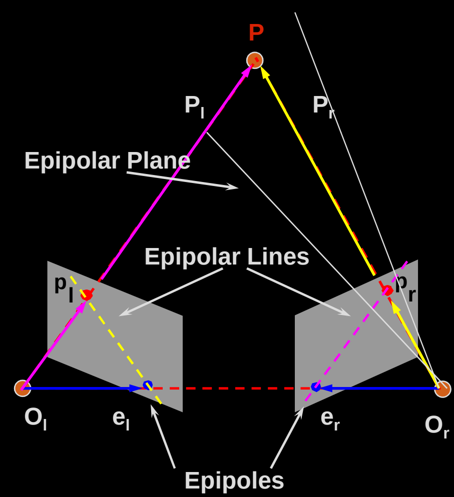
Epipolar Geometry

- Motivation: where to search correspondences?
  - Epipolar Plane
    - A plane going through point P and the centers of projections (COPs) of the two cameras
  - Conjugated Epipolar Lines
    - Lines where epipolar plane intersects the image planes
  - Epipoles
    - The image of the COP of one camera in the other
- Epipolar Constraint
  - Corresponding points must lie on conjugated epipolar lines



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Essential Matrix

- Equation of the epipolar plane
  - Co-planarity condition of vectors P_l, T and P_l-T

$$(\mathbf{P_l} - \mathbf{T})^T \mathbf{T} \times \mathbf{P_l} = 0$$

$$\mathbf{P_r} = \mathbf{R}(\mathbf{P_l} - \mathbf{T})$$

- Essential Matrix E = RS
  - 3x3 matrix constructed from R and T (extrinsic only)
    - Rank (E) = 2, two equal nonzero singular values

$$\mathbf{R} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$$

Rank (R) =3

$$S = \begin{bmatrix} 0 & -T_z & T_y \\ T_z & 0 & -T_x \\ -T_y & T_x & 0 \end{bmatrix}$$

Rank (S) =2

$$\mathbf{P_r}^{\mathbf{T}} \mathbf{E} \mathbf{P_l} = 0$$

$$\mathbf{p}_l = \frac{f_l}{Z_l} \mathbf{P_l} \qquad \mathbf{p}_r = \frac{f_r}{Z_r} \mathbf{P}_r$$

$$\mathbf{p_r}^{\mathbf{T}} \mathbf{E} \mathbf{p_l} = 0$$

Essential Matrix

- Essential Matrix $E = RS$ $\mathbf{p_r}^T \mathbf{E} \mathbf{p_l} = 0$
  - A natural link between the stereo point pair and the extrinsic parameters of the stereo system
    - One correspondence -> a linear equation of 9 entries
    - Given 8 pairs of (pl, pr) -> E
  - Mapping between points and epipolar lines we are looking for
    - Given p_l, E -> p_r on the projective line in the right plane
    - Equation represents the epipolar line of pr (or pl) in the right (or left) image
- Note:
  - pl, pr are in the camera coordinate system, not pixel coordinates that we can measure

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Fundamental Matrix

- Mapping between points and epipolar lines in the pixel coordinate systems
  - With no prior knowledge on the stereo system
- From Camera to Pixels: Matrices of intrinsic parameters

$$\mathbf{M}_{int} = \begin{bmatrix} -f_x & 0 & o_x \\ 0 & -f_y & o_y \\ 0 & 0 & 1 \end{bmatrix}$$

Rank (M_{int}) =3

$$\mathbf{p_r}^T \mathbf{E} \mathbf{p_l} = 0$$

$$\mathbf{p_l} = \mathbf{M_l}^{-1} \overline{\mathbf{p}}_\mathbf{l} \qquad \mathbf{p}_r = \mathbf{M}_r^{-1} \overline{\mathbf{p}}_r$$

$$\overline{\mathbf{p}}_\mathbf{r}^T \mathbf{F} \overline{\mathbf{p}}_\mathbf{l} = 0$$

$$\mathbf{F} = \mathbf{M}_r^{-T} \mathbf{E} \mathbf{M}_l^{-1}$$

- Questions:
  - What are fx, fy, ox, oy ?
  - How to measure $\overline{p}_l$ in images?

Fundamental Matrix

- Fundamental Matrix
  - Rank (F) = 2
  - Encodes info on both intrinsic and extrinsic parameters

$$\mathbf{F} = \mathbf{M}_r^{-T}\mathbf{E}\mathbf{M}_l^{-1}$$

  - Enables full reconstruction of the epipolar geometry
  - In pixel coordinate systems without any knowledge of the intrinsic and extrinsic parameters
  - Linear equation of the 9 entries of F

$$\bar{\mathbf{p}}_\mathbf{r}^\mathbf{T}\mathbf{F}\bar{\mathbf{p}}_\mathbf{l} = 0 \Rightarrow (x_{im}^{(r)} \quad y_{im}^{(r)} \quad 1)\begin{bmatrix} f11 & f12 & f13 \\ f21 & f22 & f23 \\ f31 & f32 & f33 \end{bmatrix}\begin{pmatrix} x_{im}^{(l)} \\ y_{im}^{(l)} \\ 1 \end{pmatrix} = 0$$

Computing F: The Eight-point Algorithm

- Input: n point correspondences (n >= 8)
  - Construct homogeneous system Ax= 0 from $\bar{\mathbf{p}}_\mathbf{r}^\mathbf{T}\mathbf{F}\bar{\mathbf{p}}_\mathbf{l} = 0$
    - x = (f_{11},f_{12}, ,f_{13}, f_{21},f_{22},f_{23} f_{31},f_{32}, f_{33}) : entries in F
    - Each correspondence give one equation
    - A is a nx9 matrix
  - Obtain estimate F^ by SVD of A $\mathbf{A} = \mathbf{U}\mathbf{D}\mathbf{V}^T$
    - x (up to a scale) is column of V corresponding to the least singular value
  - Enforce singularity constraint: since Rank (F) = 2
    - Compute SVD of F^ $\hat{\mathbf{F}} = \mathbf{U}\mathbf{D}\mathbf{V}^T$
    - Set the smallest singular value to 0: D -> D'
    - Correct estimate of F : $\mathbf{F}' = \mathbf{U}\mathbf{D}'\mathbf{V}^T$
- Output: the estimate of the fundamental matrix, F'
- Similarly we can compute E given intrinsic parameters

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Locating the Epipoles from F

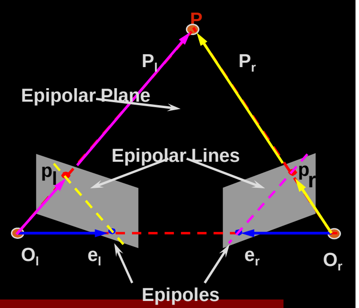
$\bar{\mathbf{p}}_r^{\,T} \mathbf{F} \bar{\mathbf{p}}_l = 0$ e_l lies on all the epipolar lines of the left image

$\Downarrow$

$\bar{\mathbf{p}}_r^{\,T} \mathbf{F} \bar{\mathbf{e}}_l = 0$ **For every p_r**

$\Downarrow$ F is not identically zero

$\mathbf{F} \bar{\mathbf{e}}_l = 0$



- Input: Fundamental Matrix F
  - Find the SVD of F
  - The epipole e_l is the column of V corresponding to the null singular value (as shown above)
  - The epipole e_r is the column of U corresponding to the null singular value
- Output: Epipole e_l and e_r

$$\mathbf{F} = \mathbf{U}\mathbf{D}\mathbf{V}^T$$

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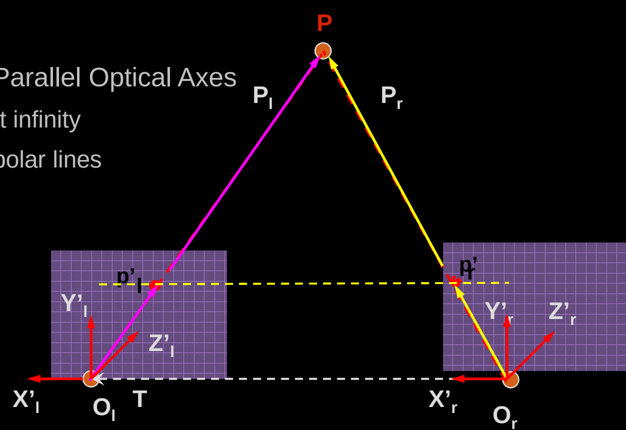
Break

- Homework #4 online, due on November 29 before class

Stereo Rectification

- Stereo System with Parallel Optical Axes
  - Epipoles are at infinity
  - Horizontal epipolar lines



- Rectification
  - Given a stereo pair, the intrinsic and extrinsic parameters, find the image transformation to achieve a stereo system of horizontal epipolar lines
  - A simple algorithm: Assuming calibrated stereo cameras

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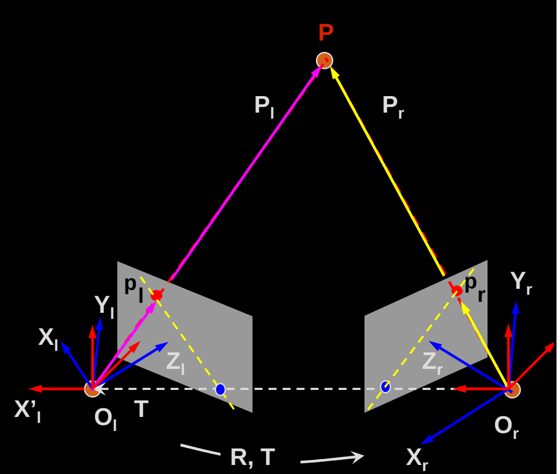
Stereo Rectification

- Algorithm
  - Rotate both left and right camera so that they share the same X axis : O_r-O_l = T
  - Define a rotation matrix R_{rect} for the left camera
  - Rotation Matrix for the right camera is $R_{rect}R^T$
  - Rotation can be implemented by image transformation



$X_l' = T$, $Y_l' = X_l' \times Z_l$, $Z'_l = X_l' \times Y_l'$

Stereo Rectification

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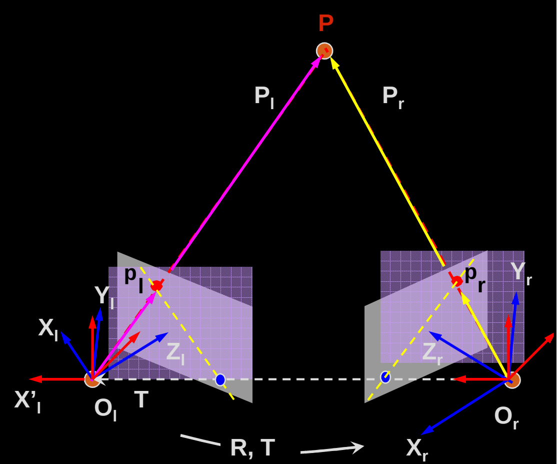

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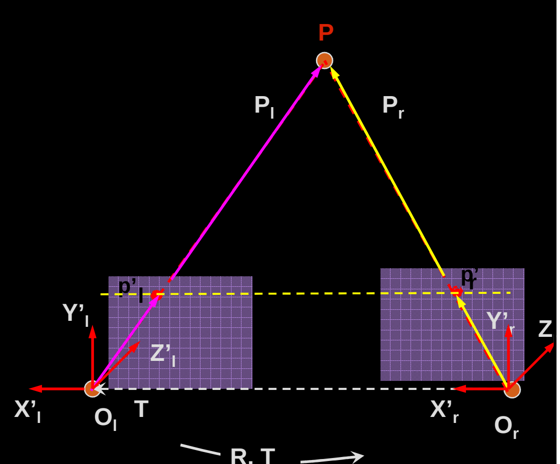

$T' = (B, 0, 0)$, $P'_r = P'_l - T'$

Epipolar Geometry: Summary

- Purpose
  - where to search correspondences
- Epipolar plane, epipolar lines, and epipoles
  - known intrinsic (f) and extrinsic (R, T)
    - co-planarity equation
  - known intrinsic but unknown extrinsic
    - essential matrix
  - unknown intrinsic and extrinsic
    - fundamental matrix
- Rectification
  - Generate stereo pair (by software) with parallel optical axis and thus horizontal epipolar lines

$$\mathbf{P}_r^T \mathbf{R}^T \mathbf{T} \times \mathbf{P}_l = 0$$

$$\mathbf{p_r}^{\mathbf{T}} \mathbf{E} \mathbf{p_l} = 0$$

$$\overline{\mathbf{p}}_{\mathbf{r}}^{\mathbf{T}} \mathbf{F} \overline{\mathbf{p}}_{\mathbf{l}} = 0$$

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Part II. Correspondence problem

- Three Questions
  - What to match?
    - Features: point, line, area, structure?
  - Where to search correspondence?
    - Epipolar line?
  - How to measure similarity?
    - Depends on features
- Approaches
  - Correlation-based approach
  - Feature-based approach
- Advanced Topics
  - Image filtering to handle illumination changes
  - Adaptive windows to deal with multiple disparities
  - Local warping to account for perspective distortion
  - Sub-pixel matching to improve accuracy
  - Self-consistency to reduce false matches
  - Multi-baseline stereo

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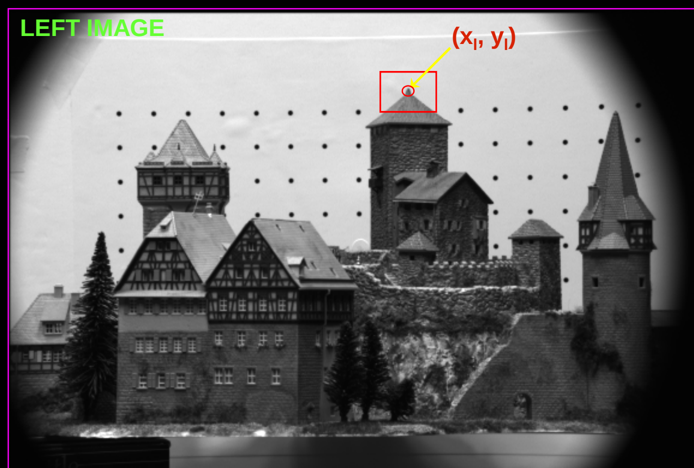
Correlation Approach



- For Each point (x_l, y_l) in the left image, define a window centered at the point

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Correlation Approach

- … search its corresponding point within a search region in the right image

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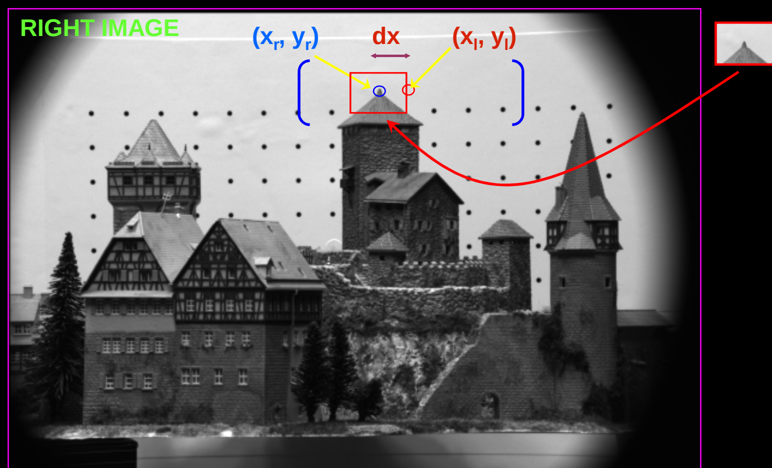
Correlation Approach



- … the disparity (dx, dy) is the displacement when the correlation is maximum

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Correlation Approach

- Elements to be matched
  - Image window of fixed size centered at each pixel in the left image
- Similarity criterion
  - A measure of similarity between windows in the two images
  - The corresponding element is given by window that maximizes the similarity criterion within a search region
- Search regions
  - Theoretically, search region can be reduced to a 1-D segment, along the epipolar line, and within the disparity range.
  - In practice, search a slightly larger region due to errors in calibration

Correlation Approach

- Equations

$$c(dx,dy) = \sum_{k=-W}^{W} \sum_{l=-W}^{W} \psi(I_l(x_l + k, y_l + l), I_r(x_l + dx + k, y_l + dy + l))$$

- disparity

$$\bar{\mathbf{d}} = (\bar{dx}, \bar{dy}) = \arg\max_{\mathbf{d} \in R} \{c(dx,dy)\}$$

- Similarity criterion
  - Cross-Correlation $\Psi(u,v) = uv$
  - Sum of Square Difference (SSD) $\Psi(u,v) = -(u-v)^2$
  - Sum of Absolute Difference(SAD) $\Psi(u,v) = -|u-v|$

Correlation Approach

- PROS
  - Easy to implement
  - Produces dense disparity map
  - Maybe slow
- CONS
  - Needs textured images to work well
  - Inadequate for matching image pairs from very different viewpoints due to illumination changes
  - Window may cover points with quite different disparities
  - Inaccurate disparities on the occluding boundaries

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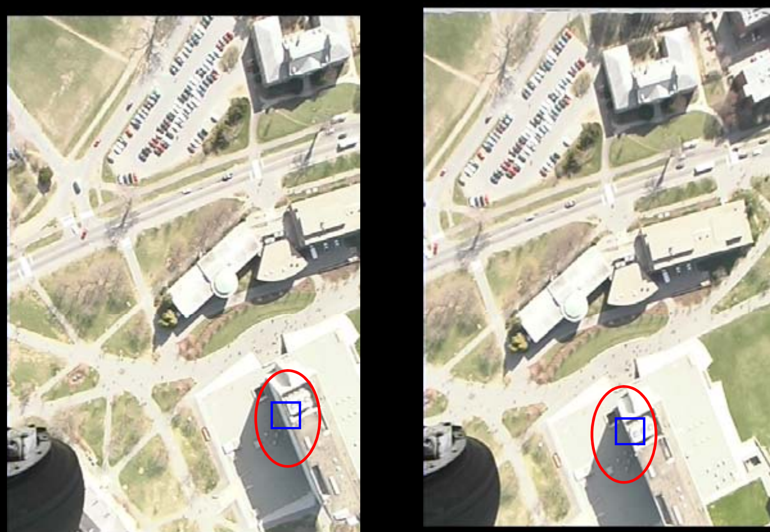
Correlation Approach

- A Stereo Pair of UMass Campus – texture, boundaries and occlusion



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Feature-based Approach

- Features
  - Edge points
  - Lines (length, orientation, average contrast)
  - Corners
- Matching algorithm
  - Extract features in the stereo pair
  - Define similarity measure
  - Search correspondences using similarity measure and the epipolar geometry

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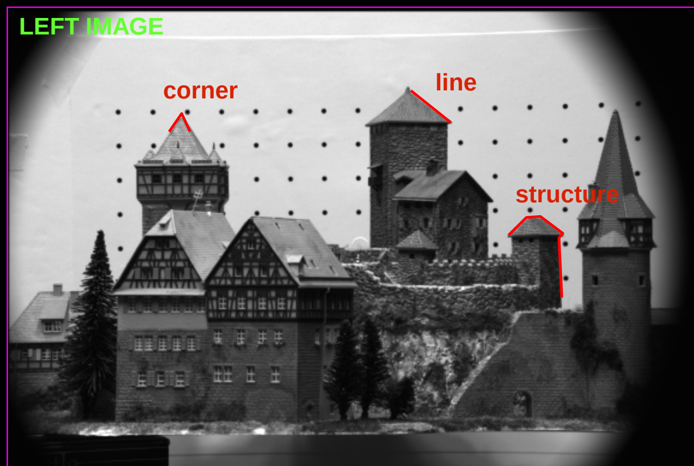
Feature-based Approach



- For each feature in the left image…

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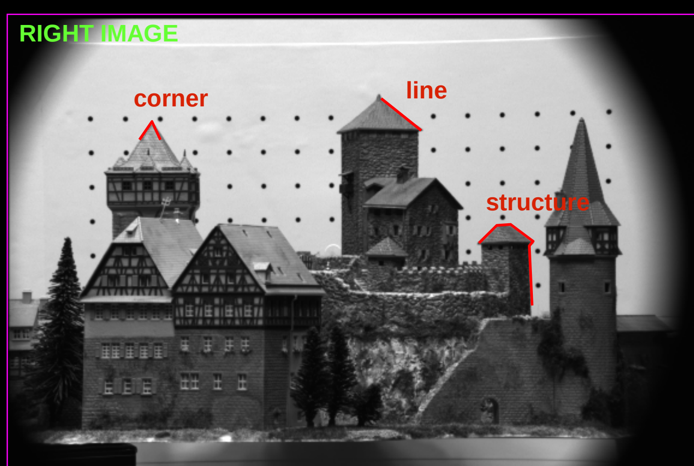
Feature-based Approach



- Search in the right image… the disparity (dx, dy) is the displacement when the similarity measure is maximum

Feature-based Approach

- PROS
  - Relatively insensitive to illumination changes
  - Good for man-made scenes with strong lines but weak texture or textureless surfaces
  - Work well on the occluding boundaries (edges)
  - Could be faster than the correlation approach
- CONS
  - Only sparse depth map
  - Feature extraction may be tricky
    - Lines (Edges) might be partially extracted in one image
    - How to measure the similarity between two lines?

Break

- Homework #4 online, due on November 29 before class

Advanced Topics

- Mainly used in correlation-based approach, but can be applied to feature-based match
- Image filtering to handle illumination changes
  - Image equalization
    - To make two images more similar in illumination
  - Laplacian filtering (2nd order derivative)
    - Use derivative rather than intensity (or original color)

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Advanced Topics

- Adaptive windows to deal with multiple disparities
  - Adaptive Window Approach (Kanade and Okutomi)
    - statistically adaptive technique which selects at each pixel the window size that minimizes the uncertainty in disparity estimates
    - **A Stereo Matching Algorithm with an Adaptive Window: Theory and Experiment**, *T. Kanade and M. Okutomi. Proc. 1991 IEEE International Conference on Robotics and Automation*, Vol. 2, April, 1991, pp. 1088-1095
  - Multiple window algorithm (Fusiello, et al)
    - Use 9 windows instead of just one to compute the SSD measure
    - The point with the smallest SSD error amongst the 9 windows and various search locations is chosen as the best estimate for the given points
    - A Fusiello, V. Roberto and E. Trucco, Efficient stereo with multiple windowing, IEEE CVPR pp858-863, 1997

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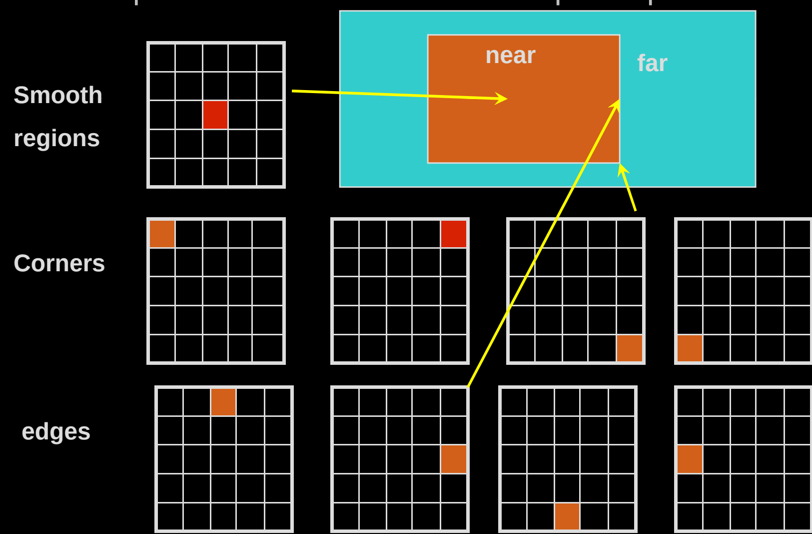
Advanced Topics

- Multiple windows to deal with multiple disparities



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Advanced Topics

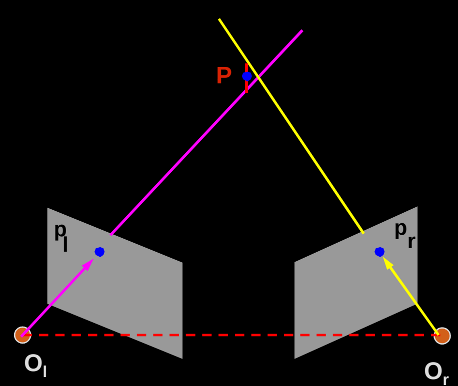
- Sub-pixel matching to improve accuracy
  - Find the peak in the correlation curves
- Self-consistency to reduce false matches esp. for occlusions
  - Check the consistency of matches from L to R and from R to L
- Multiple Resolution Approach
  - From coarse to fine for efficiency in searching correspondences
- Local warping to account for perspective distortion
  - Warp from one view to the other for a small patch given an initial estimation of the (planar) surface normal
- Multi-baseline Stereo
  - Improves both correspondences and 3D estimation by using more than two cameras (images)

3D Reconstruction Problem

- What we have done
  - Correspondences using either correlation or feature based approaches
  - Epipolar Geometry from at least 8 point correspondences
- Three cases of 3D reconstruction depending on the amount of a priori knowledge on the stereo system
  - Both intrinsic and extrinsic known - > can solve the reconstruction problem unambiguously by triangulation
  - Only intrinsic known -> recovery structure and extrinsic up to an unknown scaling factor
  - Only correspondences -> reconstruction only up to an unknown, global projective transformation (*)

Reconstruction by Triangulation

- Assumption and Problem
  - Under the assumption that both intrinsic and extrinsic parameters are known
  - Compute the 3-D location from their projections, pl and pr
- Solution
  - Triangulation: Two rays are known and the intersection can be computed
  - Problem: Two rays will not actually intersect in space due to errors in calibration and correspondences, and pixelization
  - Solution: find a point in space with minimum distance from both rays



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Reconstruction up to a Scale Factor

- Assumption and Problem Statement
  - Under the assumption that only intrinsic parameters and more than 8 point correspondences are given
  - Compute the 3-D location from their projections, pl and pr, as well as the extrinsic parameters
- Solution
  - Compute the essential matrix E from at least 8 correspondences
  - Estimate T (up to a scale and a sign) from E (=RS) using the orthogonal constraint of R, and then R
    - End up with four different estimates of the pair (T, R)
  - Reconstruct the depth of each point, and pick up the correct sign of R and T.
  - Results: reconstructed 3D points (up to a common scale);
  - The scale can be determined if distance of two points (in space) are known

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Reconstruction up to a Projective Transformation

(* not required for this course; needs advanced knowledge of projective geometry)

- Assumption and Problem Statement
  - Under the assumption that only n (>=8) point correspondences are given
  - Compute the 3-D location from their projections, pl and pr
- Solution
  - Compute the Fundamental matrix F from at least 8 correspondences, and the two epipoles
  - Determine the projection matrices
    - Select five points (from correspondence pairs) as the projective basis
  - Compute the projective reconstruction
    - Unique up to the unknown projective transformation fixed by the choice of the five points

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Summary

- Fundamental concepts and problems of stereo
- Epipolar geometry and stereo rectification
- Estimation of fundamental matrix from 8 point pairs
- Correspondence problem and two techniques: correlation and feature based matching
- Reconstruct 3-D structure from image correspondences given
  - Fully calibrated
  - Partially calibration
  - Uncalibrated stereo cameras (*)

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Next

- Understanding 3D structure and events from motion

Motion

- Homework #4 online, due on November 29 before class