3D Computer Vision
and Video Computing

3D Vision

CSc I6716
Fall 2010

Topic 2 of Part II
Calibration

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Lecture Outline

- Calibration: Find the intrinsic and extrinsic parameters
  - Problem and assumptions
  - Direct parameter estimation approach
  - Projection matrix approach

- Direct Parameter Estimation Approach
  - Basic equations (from Lecture 5)
  - Homogeneous System
  - Estimating the Image center using vanishing points
  - SVD (Singular Value Decomposition)
  - Focal length, Aspect ratio, and extrinsic parameters
  - Discussion: Why not do all the parameters together?

- Projection Matrix Approach (…after-class reading)
  - Estimating the projection matrix M
  - Computing the camera parameters from M
  - Discussion

- Comparison and Summary
  - Any difference?
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Problem and Assumptions

- Given one or more images of a calibration pattern,
- Estimate
  - The intrinsic parameters
  - The extrinsic parameters, or
  - BOTH

- Issues: Accuracy of Calibration
  - How to design and measure the calibration pattern
    - Distribution of the control points to assure stability of solution – not coplanar
    - Construction tolerance one or two order of magnitude smaller than the desired accuracy of calibration
    - e.g. 0.01 mm tolerance versus 0.1mm desired accuracy
  - How to extract the image correspondences
    - Corner detection?
    - Line fitting?
  - Algorithms for camera calibration given both 3D-2D pairs

- Alternative approach: 3D from un-calibrated camera

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Camera Model

- Coordinate Systems
  - Frame coordinates \((x_{im}, y_{im})\) pixels
  - Image coordinates \((x, y)\) in mm
  - Camera coordinates \((X, Y, Z)\)
  - World coordinates \((X_w, Y_w, Z_w)\)

- Camera Parameters
  - Intrinsic Parameters (of the camera and the frame grabber): link the frame coordinates of an image point with its corresponding camera coordinates
  - Extrinsic parameters: define the location and orientation of the camera coordinate system with respect to the world coordinate system
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Linear Version of Perspective Projection

- World to Camera
  - Camera: \( P = (X, Y, Z)^T \)
  - World: \( P_w = (X_w, Y_w, Z_w)^T \)
  - Transform: \( R, T \)

- Camera to Image
  - Camera: \( P = (X, Y, Z)^T \)
  - Image: \( p = (x, y)^T \)
  - Not linear equations

- Image to Frame
  - Neglecting distortion
  - Frame \((x_{im}, y_{im})^T\)

- World to Frame
  - \((X_w, Y_w, Z_w)^T \rightarrow (x_{im}, y_{im})^T\)
  - Effective focal lengths
    - \( f = f/x, f_y = f/y \)

Direct Parameter Method

- Extrinsic Parameters
  - \( R \), 3x3 rotation matrix
  - \( T \), 3-D translation vector
  - Three angles \( \alpha, \beta, \gamma \)

- Intrinsic Parameters
  - \( f_x, f_y \) : effective focal length in pixel
  - \( \alpha = f_x/f_y = s_y/s_x \), and \( fx \)
  - \( (ox, oy) \): known image center \( \rightarrow (x, y) \) known
  - \( k_1 \), radial distortion coefficient: neglect it in the basic algorithm

- Same Denominator in the two Equations
  - Known: \((X_w, Y_w, Z_w)\) and its \((x, y)\)
  - Unknown: \( r_{pq}, T_x, T_y, f_x, f_y \)

\[
\frac{f_y (r_{21} X_w + r_{22} Y_w + r_{23} Z_w + T_y)}{f_x (r_{11} X_w + r_{12} Y_w + r_{13} Z_w + T_x)} = x', \quad \frac{y'}{x'} = f_y (r_{11} X_w + r_{12} Y_w + r_{13} Z_w + T_x) / f_x (r_{21} X_w + r_{22} Y_w + r_{23} Z_w + T_y)
\]

\[
x' = x_{im} - o_x x, \quad y' = y_{im} - o_y y
\]
Linear Equations

- Linear Equation of 8 unknowns $\mathbf{v} = (v_1, \ldots, v_8)$
  - Aspect ratio: $\alpha = \frac{f_x}{f_y}$
  - Point pairs $(X_i, Y_i, Z_i) \leftrightarrow (x_i, y_i)$ drop the ' and subscript "w"

$$x'(r_1X_w + r_2Y_w + r_3Z_w + T_w) = y'\alpha(r_1X_w + r_2Y_w + r_3Z_w + T_w)$$

$x_iX_1r_21 + x_iY_1r_22 + x_iZ_1r_23 + x_iT_y - y_iX_1(\alpha r_{11}) - y_iY_1(\alpha r_{12}) - y_iZ_1(\alpha r_{13}) - y_i(\alpha T_x) = 0$

$$x_iX_1v_1 + x_iY_1v_2 + x_iZ_1v_3 + x_iy_4 - y_iX_1v_5 - y_iY_1v_6 - y_iZ_1v_7 - y_iy_8 = 0$$

Homogeneous System

- Homogeneous System of N Linear Equations
  - Given N corresponding pairs $((X_i, Y_i, Z_i) \leftrightarrow (x_i, y_i), i=1,2,\ldots N$
  - 8 unknowns $\mathbf{v} = (v_1, \ldots, v_8)^T$, 7 independent parameters

$$x_iX_1v_1 + x_iY_1v_2 + x_iZ_1v_3 + x_iy_4 - y_iX_1v_5 - y_iY_1v_6 - y_iZ_1v_7 - y_iy_8 = 0$$

$$\mathbf{A} \mathbf{v} = 0$$

- The system has a nontrivial solution (up to a scale)
  - IF $N >= 7$ and N points are not coplanar $\Rightarrow$ Rank $(\mathbf{A}) = 7$
  - Can be determined from the SVD of $\mathbf{A}$
Homework #3 online, due October 25 (Monday) before class

Homogeneous System

- Homogeneous System of N Linear Equations
  - Given N corresponding pairs \{ (X_i, Y_i, Z_i) <-> (x_i, y_i) \}, i=1,2,…N
  - 8 unknowns \( \mathbf{v} = (v_1, \ldots, v_8)^T \), 7 independent parameters
  
  \[
  x_i X_i v_1 + x_i Y_i v_2 + x_i Z_i v_3 + x_i v_4 - y_i X_i v_5 - y_i Y_i v_6 - y_i Z_i v_7 - y_i v_8 = 0
  \]

  \[
  \mathbf{A} \mathbf{v} = 0
  \]

  - The system has a nontrivial solution (up to a scale)
    - IF N >= 7 and N points are not coplanar => Rank (\( \mathbf{A} \)) = 7
    - Can be determined from the SVD of \( \mathbf{A} \)
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SVD: definition

- Singular Value Decomposition:
  - Any mxn matrix can be written as the product of three
    matrices

\[
A = UDV^T
\]

- Singular values \( \sigma_i \) are fully determined by \( A \)
  - \( D \) is diagonal: \( d_{ij} = 0 \) if \( i \neq j; d_{ii} = \sigma_i \) (\( i = 1, 2, \ldots, n \))
  - \( \sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_n \geq 0 \)
  - Both \( U \) and \( V \) are not unique
  - Columns of each are mutual orthogonal vectors

Appendix A.6

SVD: properties

- 1. Singularity and Condition Number
  - \( nxn \) A is nonsingular IFF all singular values are nonzero
  - Condition number: degree of singularity of \( A \)
    \( C = \sigma_1 / \sigma_n \)
    - \( A \) is ill-conditioned if \( 1/C \) is comparable to the arithmetic
      precision of your machine; almost singular

- 2. Rank of a square matrix \( A \)
  - Rank \( (A) = \) number of nonzero singular values

- 3. Inverse of a square Matrix
  - If \( A \) is nonsingular
    \( A^{-1} = VD^{-1}U^T \)
  - In general, the pseudo-inverse of \( A \)
    \( A^+ = VD_0^{-1}U^T \)

- 4. Eigenvalues and Eigenvectors (questions)
  - Eigenvalues of both \( A^TA \) and \( AA^T \) are \( \sigma_i^2 \) (\( \sigma_i > 0 \))
  - The columns of \( U \) are the eigenvectors of \( AA^T \) (\( mxm \))
  - The columns of \( V \) are the eigenvectors of \( A^TA \) (\( nxn \))

\[
\begin{align*}
AA^T u_i &= \sigma_i^2 u_i \\
A^T A v_j &= \sigma_j^2 v_j
\end{align*}
\]
### SVD: Application 1

**Least Square**
- Solve a system of $m$ equations for $n$ unknowns $\mathbf{x}(m \geq n)$
- $\mathbf{A}$ is a $m \times n$ matrix of the coefficients
- $\mathbf{b}$ ($\neq 0$) is the $m$-D vector of the data
- Solution:
  $$\mathbf{A}^T \mathbf{A} \mathbf{x} = \mathbf{A}^T \mathbf{b}$$
  $$\mathbf{x} = (\mathbf{A}^T \mathbf{A})^+ \mathbf{A}^T \mathbf{b}$$

- How to solve: compute the pseudo-inverse of $\mathbf{A}^T \mathbf{A}$ by SVD
  - $(\mathbf{A}^T \mathbf{A})^+$ is more likely to coincide with $(\mathbf{A}^T \mathbf{A})^{-1}$ given $m > n$
  - Always a good idea to look at the condition number of $\mathbf{A}^T \mathbf{A}$

### SVD: Application 2

**Homogeneous System**
- $m$ equations for $n$ unknowns $\mathbf{x}(m \geq n-1)$
- Rank $(\mathbf{A}) = n-1$ (by looking at the SVD of $\mathbf{A}$)
- A non-trivial solution (up to an arbitrary scale) by SVD:
  - Simply proportional to the eigenvector corresponding to the only zero eigenvalue of $\mathbf{A}^T \mathbf{A}$ ($n \times n$ matrix)
  - Note:
    - All the other eigenvalues are positive because Rank $(\mathbf{A}) = n-1$
    - For a proof, see Textbook p. 324-325
    - In practice, the eigenvector (i.e. $\mathbf{v}_n$) corresponding to the minimum eigenvalue of $\mathbf{A}^T \mathbf{A}$, i.e. $\sigma_n^2$
Problem Statements
- Numerical estimate of a matrix $A$ whose entries are not independent
- Errors introduced by noise alter the estimate to $\hat{A}$

Enforcing Constraints by SVD
- Take orthogonal matrix $A$ as an example
- Find the closest matrix to $\hat{A}$, which satisfies the constraints exactly
  - SVD of $\hat{A}$: $\hat{A} = UDV^T$
  - Observation: $D = I$ (all the singular values are 1) if $A$ is orthogonal
  - Solution: changing the singular values to those expected
    $$A = UIV^T$$

Homogeneous System
- Homogeneous System of $N$ Linear Equations
  - Given $N$ corresponding pairs $\{(X_i, Y_i, Z_i) \leftrightarrow (x_i, y_i)\}$, $i=1,2,...,N$
  - 8 unknowns $v = (v_1, v_8)^T$, 7 independent parameters
  - The system has a nontrivial solution (up to a scale)
    - IF $N \geq 7$ and $N$ points are not coplanar $\Rightarrow$ Rank ($A$) = 7
    - Can be determined from the SVD of $A$
    - Rows of $V^T$: eigenvectors $\{e_i\}$ of $A^TA$
    - Solution: the $8^{th}$ row $e_8$ corresponding to the only zero singular value $\lambda_8 = 0$
      $$v = ce_8$$
  - Practical Consideration
    - The errors in localizing image and world points may make the rank of $A$ to be maximum (8)
    - In this case select the eigenvector corresponding to the smallest eigenvalue.
Scale Factor and Aspect Ratio

Equations for scale factor $\gamma$ and aspect ratio $\alpha$

$\vec{V} = \gamma \left( r_{21}, r_{22}, r_{23}, T_y, \alpha r_{11}, \alpha r_{12}, \alpha r_{13}, \alpha T_x \right)$

$\begin{bmatrix} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 & v_8 \end{bmatrix}$

Knowledge: $R$ is an orthogonal matrix

$R^T_i R_j = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}$

$R = \begin{pmatrix} \eta_1 & \eta_2 & \eta_3 \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix} = \begin{pmatrix} R_1^T \\ R_2^T \\ R_3^T \end{pmatrix}$

Second row ($i=j=2$):

$r_{21}^2 + r_{22}^2 + r_{23}^2 = 1$ $\Rightarrow$ $|\gamma| = \sqrt{v_1^2 + v_2^2 + v_3^2}$

First row ($i=j=1$):

$r_{11}^2 + r_{12}^2 + r_{13}^2 = 1$ $\Rightarrow$ $|\gamma| = \sqrt{v_1^2 + v_2^2 + v_3^2}$

Remaining Questions:
- How to find the sign $s$?
- Is $R$ orthogonal?
- How to find $T_z$ and $f_x$, $f_y$?
Facts:
- \( f_x > 0 \)
- \( Z_c > 0 \)
- \( x \) known
- \( X_w, Y_w, Z_w \) known

Solution

⇒ Check the sign of \( X_c \)
⇒ Should be opposite to \( x \)

\[
x = -f_x \frac{X_c}{Z_c} = -f_x \left( \eta_1 X_w + \eta_2 Y_w + \eta_3 Z_w + T_z \right)
\]
\[
y = -f_y \frac{Y_c}{Z_c} = -f_y \left( \eta_2 X_w + \eta_3 Y_w + \eta_3 Z_w + T_y \right)
\]

Rotation \( R \) : Orthogonality

Question:
- First 2 rows of \( R \) are calculated without using the mutual orthogonal constraint

\[
\hat{R}^T \hat{R} = I^? 
\]

Solution:
- Use SVD of estimate \( \hat{R} \)

\[
\hat{R} = UDV^T \quad \rightarrow \quad R = UIV^T 
\]

Replace the diagonal matrix \( D \) with the 3x3 identity matrix
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Find Tz, Fx and Fy

Solution
- Solve the system of N linear equations with two unknowns $T_x$, $f_x$

$$x = -f_x \left( \frac{\eta_1 X_w + \eta_2 Y_w + \eta_3 Z_w + T_z}{r_31 X_w + r_32 Y_w + r_33 Z_w + T_z} \right)$$

- Least Square method

$$\begin{bmatrix} T_z \\ f_x \end{bmatrix} = (A^T A)^{-1} A^T b$$

- SVD method to find inverse

Direct Parameter Calibration Summary

Algorithm (p130-131)

1. Measure N 3D coordinates $(X_i, Y_i, Z_i)$
2. Locate their corresponding image points $(x_i, y_i)$ - Edge, Corner, Hough
3. Build matrix $A$ of a homogeneous system $Av = 0$
4. Compute SVD of $A$, solution $v$
5. Determine aspect ratio $\alpha$ and scale $|\gamma|$
6. Recover the first two rows of $R$ and the first two components of $T$ up to a sign
7. Determine sign $s$ of $\gamma$ by checking the projection equation
8. Compute the 3rd row of $R$ by vector product, and enforce orthogonality constraint by SVD
9. Solve $T_z$ and $f_x$ using Least Square and SVD, then $f_y = f_x / \alpha$
Homework #3 online, due October 25 before class

Questions

- Can we select an arbitrary image center for solving other parameters?
- How to find the image center (ox, oy)?
- How about to include the radial distortion?
- Why not solve all the parameters once?
  - How many unknown with ox, oy? --- 20 ??? – projection matrix method

\[
\begin{align*}
x &= x_{im} - o_x = -f_x \left( \frac{r_{11}X_w + r_{12}Y_w + r_{13}Z_w + T_x}{r_{31}X_w + r_{32}Y_w + r_{33}Z_w + T_z} \right) \\
y &= y_{im} - o_y = -f_y \left( \frac{r_{21}X_w + r_{22}Y_w + r_{23}Z_w + T_y}{r_{31}X_w + r_{32}Y_w + r_{33}Z_w + T_z} \right)
\end{align*}
\]
Vanishing points:
- Due to perspective, all parallel lines in 3D space appear to meet in a point on the image - the vanishing point, which is the common intersection of all the image lines.
Vanishing points:
- Due to perspective, all parallel lines in 3D space appear to meet in a point on the image - the vanishing point, which is the common intersection of all the image lines.

Important property:
- Vector OV (from the center of projection to the vanishing point) is parallel to the parallel lines.
Orthocenter Theorem:

- Input: three mutually orthogonal sets of parallel lines in an image
- \( T \): a triangle on the image plane defined by the three vanishing points
- Image center = orthocenter of triangle \( T \)
- Orthocenter of a triangle is the common intersection of the three altitudes
Orthocenter Theorem:
- Input: three mutually orthogonal sets of parallel lines in an image
- $T$: a triangle on the image plane defined by the three vanishing points
- Image center = orthocenter of triangle $T$
- Orthocenter of a triangle is the common intersection of the three altitudes

Assumptions:
- Known aspect ratio
- Without lens distortions

Questions:
- Can we solve both aspect ratio and the image center?
- How about with lens distortions?
Direct parameter Calibration Summary

Algorithm (p130-131)
0. Estimate image center (and aspect ratio)
   1. Measure N 3D coordinates (Xi, Yi, Zi)
   2. Locate their corresponding image (xi, yi) - Edge, Corner, Hough
   3. Build matrix A of a homogeneous system
      \[ Av = 0 \]
   4. Compute SVD of A, solution v
   5. Determine aspect ratio \( \alpha \) and scale \( |\gamma| \)
   6. Recover the first two rows of R and the first two components of T up to a sign
   7. Determine sign s of \( \gamma \) by checking the projection equation
   8. Compute the 3rd row of R by vector product, and enforce orthogonality constraint by SVD
   9. Solve Tz and fx using Least Square and SVD, then \( fy = fx / \alpha \)

Remaining Issues and Possible Solution

Original assumptions:
- Without lens distortions
- Known aspect ratio when estimating image center
- Known image center when estimating others including aspect ratio

New Assumptions
- Without lens distortion
- Aspect ratio is approximately 1, or \( \alpha = fx/fy = 4:3 \); image center about \((M/2, N/2)\) given a MxN image

Solution (?)
1. Using \( \alpha = 1 \) to find image center (ox, oy)
2. Using the estimated center to find others including \( \alpha \)
3. Refine image center using new \( \alpha \); if change still significant, go to step 2; otherwise stop

Projection Matrix Approach
- Homework #3 online, due October 25 before class

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**Linear Matrix Equation of perspective projection**

- **Projective Space**
  - Add fourth coordinate
    - \( P_w = (X_w, Y_w, Z_w, 1)^T \)
  - Define \((u, v, w)^T\) such that
    - \(u/w = x_{im}, v/w = y_{im}\)
- **3x4 Matrix **\( \text{E}_{\text{ext}} \)
  - Only extrinsic parameters
  - World to camera
- **3x3 Matrix **\( \text{E}_{\text{int}} \)
  - Only intrinsic parameters
  - Camera to frame
- **Simple Matrix Product!** Projective Matrix \( M = \text{M}_{\text{int}} \text{E}_{\text{ext}} \)
  - \((X_w,Y_w,Z_w)^T -> (x_{im}, y_{im})^T\)
  - Linear Transform from projective space to projective plane
  - \(M\) defined up to a scale factor – 11 independent entries
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### Projection Matrix $M$

- **World – Frame Transform**
  - Drop “im” and “w”
  - $N$ pairs $(x_i, y_i) \leftrightarrow (X_i, Y_i, Z_i)$
  - Linear equations of $m$

- **Am = 0**

### Linear Equations of $m$

**Step 1: Estimation of projection matrix**

- **World – Frame Transform**
  - Drop “im” and “w”
  - $N$ pairs $(x_i, y_i) \leftrightarrow (X_i, Y_i, Z_i)$

- **Linear equations of $m$**
  - $2N$ equations, 11 independent variables
  - $N \geq 6$, SVD => $m$ up to a unknown scale

- **Am = 0**

- **3x4 Projection Matrix $M$**
  - Both intrinsic (4) and extrinsic (6) – 10 parameters

- **Linear equations of $m$**
  - $N \geq 6$, SVD => $m$ up to a unknown scale

- **Step 1: Estimation of projection matrix**

- **Linear equations of $m$**
  - $2N$ equations, 11 independent variables
  - $N \geq 6$, SVD => $m$ up to a unknown scale

### Mathematical Formulas

- **Projection Matrix $M$**

\[
\begin{bmatrix}
X_i \\
Y_i \\
Z_i
\end{bmatrix} = M \begin{bmatrix}
x_i \\
y_i \\
w_i
\end{bmatrix}
\]

- **Linear equations of $m$**

\[
x_i = \frac{u_i}{w_i} = \frac{m_{11}X_i + m_{12}Y_i + m_{13}Z_i + m_{14}}{m_{31}X_i + m_{32}Y_i + m_{33}Z_i + m_{34}}
\]

\[
y_i = \frac{u_i}{w_i} = \frac{m_{21}X_i + m_{22}Y_i + m_{23}Z_i + m_{24}}{m_{31}X_i + m_{32}Y_i + m_{33}Z_i + m_{34}}
\]

- **Matrix $A$**

\[
A = \begin{bmatrix}
X_1 & Y_1 & Z_1 & 1 & 0 & 0 & 0 & 0 & -x_1X_1 & -x_1Y_1 & -x_1Z_1 & -x_1 \\
0 & 0 & 0 & X_1 & Y_1 & Z_1 & 1 & -y_1X_1 & -y_1Y_1 & -y_1Z_1 & -y_1 \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots
\end{bmatrix}
\]

- **Matrix $m$**

\[
m = \begin{bmatrix}
m_{11} & m_{12} & m_{13} & m_{14} & m_{21} & m_{22} & m_{23} & m_{24} & m_{31} & m_{32} & m_{33} & m_{34}
\end{bmatrix}^T
\]
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Step 2: Computing camera parameters

- 3x4 Projection Matrix \( \mathbf{M} \)
  - Both intrinsic and extrinsic

\[
\mathbf{M} = \begin{bmatrix}
- f_x \xi_1 + o_x \xi_3 \\
- f_x \xi_2 + o_x \xi_3 \\
- f_x \xi_3 + o_x \xi_3 \\
- f_x \xi_1 + o_x \xi_3 \\
- f_x \xi_2 + o_x \xi_3 \\
- f_x \xi_3 + o_x \xi_3 \\
- f_y \xi_1 + o_y \xi_3 \\
- f_y \xi_2 + o_y \xi_3 \\
- f_y \xi_3 + o_y \xi_3 \\
\end{bmatrix}
\]

\[
\hat{\mathbf{M}} = \gamma \mathbf{M}
\]

- From \( \mathbf{M} \) to parameters (p134-135)
  - Find scale \( |\gamma| \) by using unit vector \( \mathbf{R}_3^T \)
  - Determine \( \mathbf{T}_z \) and sign of \( \gamma \) from \( m_{34} \) (i.e. \( q_{43} \))
  - Obtain \( \mathbf{R}_3^T \)
  - Find \( (O_x, O_y) \) by dot products of Rows \( q_1, q_3, q_2, q_3 \), using the orthogonal constraints of \( \mathbf{R} \)
  - Determine \( f_x \) and \( f_y \) from \( q_1 \) and \( q_2 \) (Eq. 6.19) Wrong??)
  - All the rests: \( \mathbf{R}_1^T, \mathbf{R}_2^T, \mathbf{T}_x, \mathbf{T}_y \)
  - Enforce orthognoality on \( \mathbf{R} \)?

Comparisons

- Direct parameter method and Projection Matrix method

- Properties in Common:
  - Linear system first, Parameter decomposition second
  - Results should be exactly the same

- Differences
  - Number of variables in homogeneous systems
    - Matrix method: All parameters at once, 2N Equations of 12 variables
    - Direct method in three steps: N Equations of 8 variables, N equations of 2 Variables, Image Center – maybe more stable
  - Assumptions
    - Matrix method: simpler, and more general; sometime projection matrix is sufficient so no need for parameter decompostion
    - Direct method: Assume known image center in the first two steps, and known aspect ratio in estimating image center
Guidelines for Calibration

- Pick up a well-known technique or a few
- Design and construct calibration patterns (with known 3D)
- Make sure what parameters you want to find for your camera
- Run algorithms on ideal simulated data
  - You can either use the data of the real calibration pattern or using computer generated data
  - Define a virtual camera with known intrinsic and extrinsic parameters
  - Generate 2D points from the 3D data using the virtual camera
  - Run algorithms on the 2D-3D data set
- Add noises in the simulated data to test the robustness
- Run algorithms on the real data (images of calibration target)
- If successful, you are all set
- Otherwise:
  - Check how you select the distribution of control points
  - Check the accuracy in 3D and 2D localization
  - Check the robustness of your algorithms again
  - Develop your own algorithms → NEW METHODS?

3D reconstruction using two cameras

Stereo Vision

& project discussions

- Homework #3 online, due October 25 before class