Image Resampling

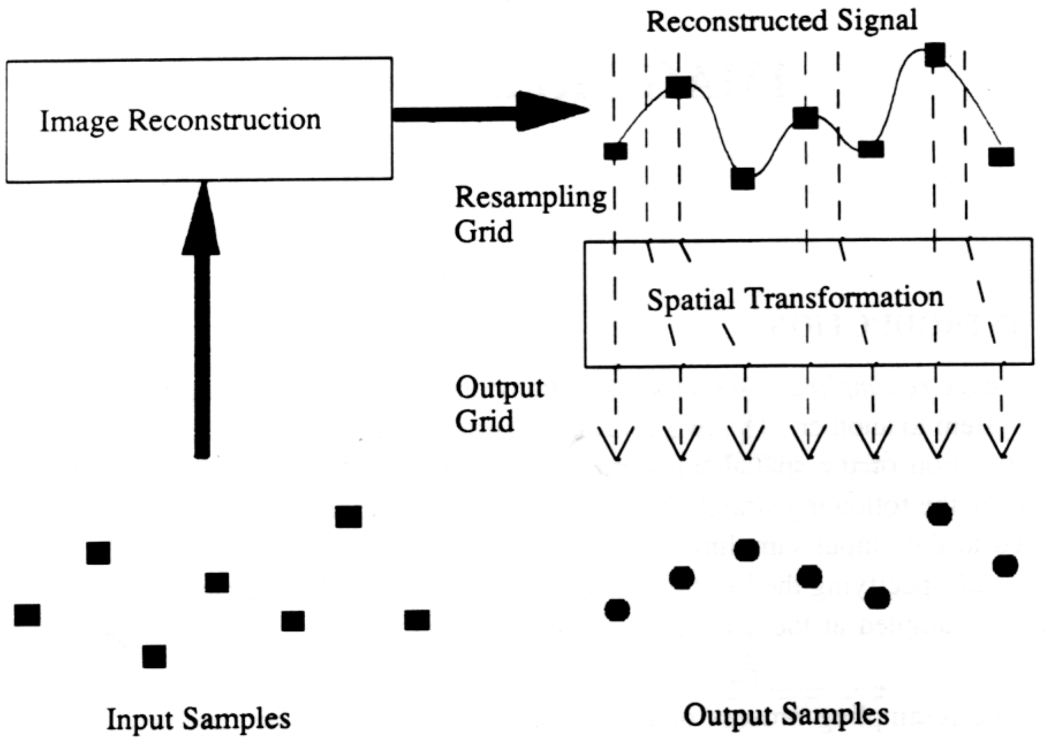
Prof. George Wolberg
Dept. of Computer Science
City College of New York

Objectives

- In this lecture we review image resampling:
  - Ideal resampling
  - Mathematical formulation
  - Resampling filter
  - Tradeoffs between accuracy and complexity
  - Software implementation

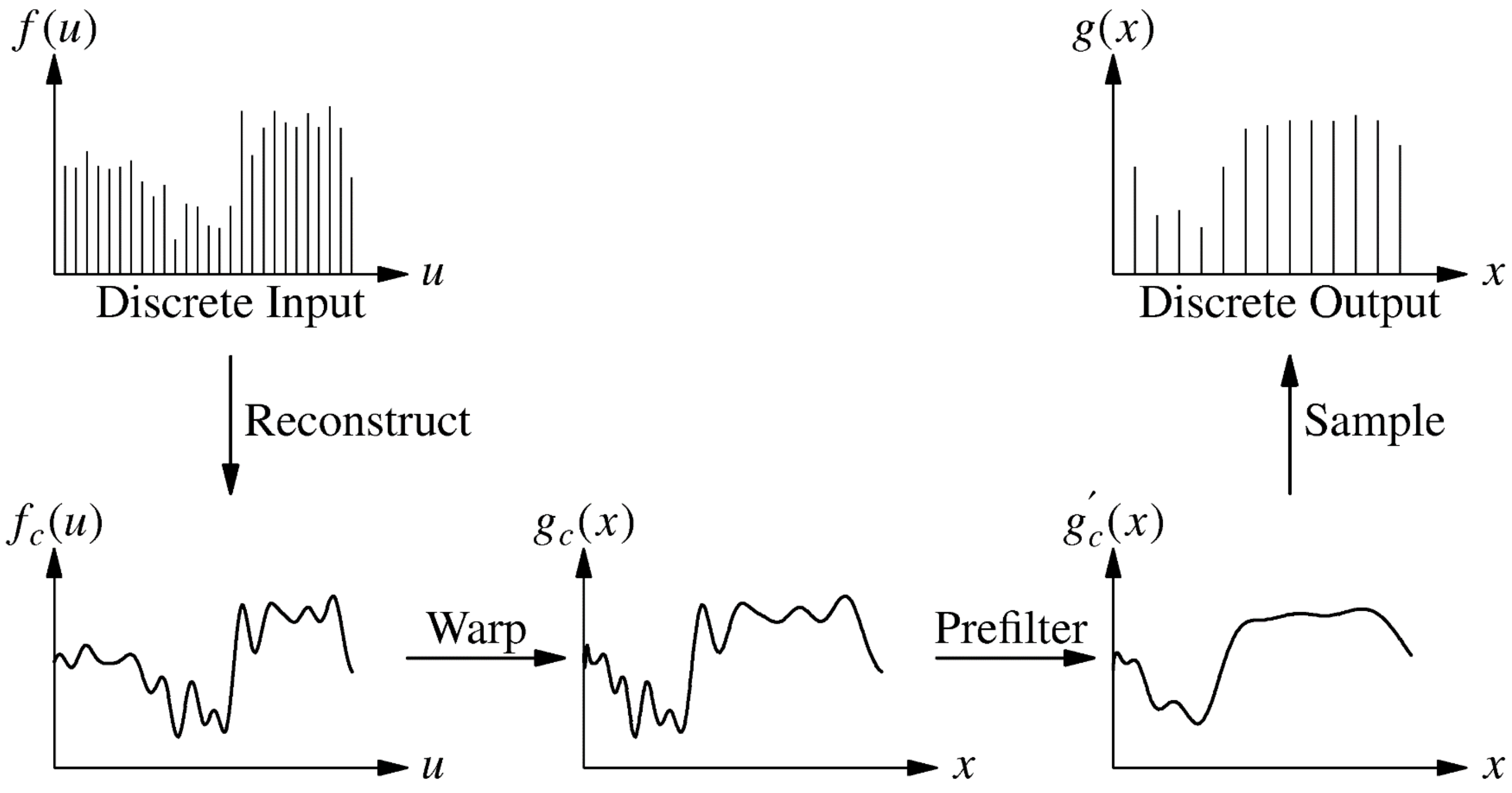
Definition

- Image Resampling: Process of transformation a sampled image from one coordinate system to another.



Ideal Resampling

- Ideal Resampling: reconstruction, warping, prefiltering, sampling



Mathematical Formulation (1)

Stages:	**Math Definition:**
Discrete Input	$f(u), u \in z$
Reconstruction Input	$f_c = f(u) * r(u) = \sum_{k \in z} f(k) r(u-k)$
Warped Signal	$g_c(x) = f_c(m^{-1}(x))$
Continuous Output	$g_c'(x) = g_c(x) * h(x) = \int g_c(t) h(x-t) dt$
Discrete Output	$g(x) = g_c'(x) S(x)$

Two filtering components: reconstruction and prefiltering (bandlimiting warped signal before sampling). Cascade them into one by working backwards from $g(x)$ to $f(u)$:

$$g(x) = g_c'(x) \qquad for \;\; x \in z$$

$$g(x) = \int f_c(m^{-1}(t)) h(x-t) dt = \int \left[\sum_{k \in z} f(k) r(m^{-1}(t) - k) \right] h(x-t) dt$$

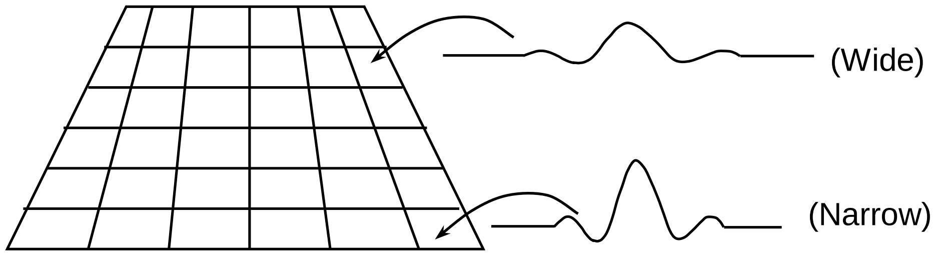
$$g(x) = \sum_{k \in z} f(k) \rho(x,k) \quad \text{where } \rho(x,k) = \int r(m^{-1}(t) - k) h(x-t) dt$$

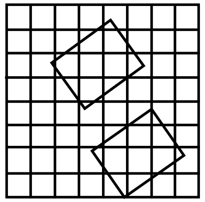
Mathematical Formulation (2)

$$\rho(x,k) = \int r(m^{-1}(t) - k)h(x-t)dt$$

Spatially varying resampling filter expressed in terms of output space

Selects filter response used to index filter coefficients





Mathematical Formulation (3)

- We can express $\rho(x,k)$ in terms of input space:

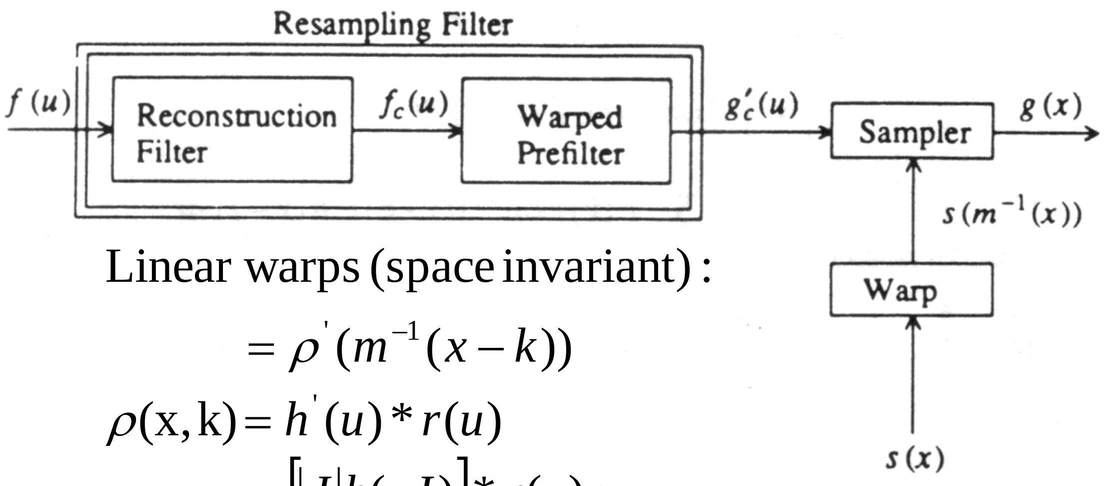
Let $t = m(u)$, we have

$$\rho(x,k) = \int r(u-k)h(x-m(u))\left|\frac{\partial m}{\partial u}\right| dt$$

$$\text{where } \left|\frac{\partial m}{\partial u}\right| \text{ is the determinant of Jacobian matrix :}$$

$$\text{1D} : \left|\frac{\partial m}{\partial u}\right| = \frac{dm}{du} \qquad \text{2D} : \left|\frac{\partial m}{\partial u}\right| = \begin{vmatrix} x_u & x_v \\ y_u & y_v \end{vmatrix} \text{ where } x_u = \frac{\partial x}{\partial u}$$

Resampling Filter



Linear warps (space invariant):

$$\rho(\mathrm{x,k}) = h'(u) * r(u) = \rho'(m^{-1}(x-k))$$

$$= \left[|J| h(uJ)\right] * r(u)$$

$\rho_{mag}(x,k) = r(u)$: Shape of *r(u)* remains the same, independently of *m(u)*, (independently of scale factor)

$\rho_{min}(x,k) = |J|h(uJ)$: Shape of prefilter is based on desired frequency response characteristic (performance in passband and stopband). Unlike *r(u)*, though, the prefilter must be scaled proportional to the minification factor (broader and shorter for more minification)

Fourier Transform Pairs

- The shape of ρ_{min} is a direct consequence of the reciprocal relation between the spatial and frequency domains.

$$\leftrightarrow \; denotes\; Fourier\; Transform\; pair$$

$$H(u) = \int h(u) e^{-i2\pi fu} du$$

$$H(m(u)) = \int h(m(u)) e^{-i2\pi fu} du$$

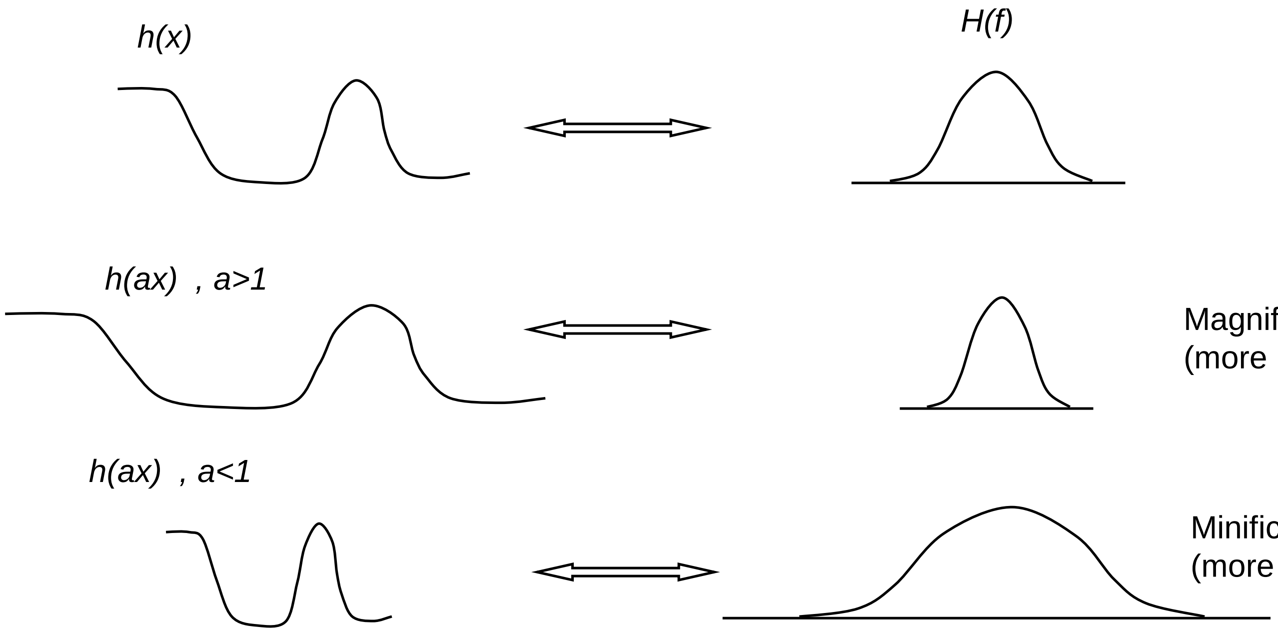
$$Let\; x = au = m(u)\; and\; dx = \left|\frac{\partial m}{\partial u}\right| du$$

$$H(m(u)) = \int h(x) e^{-i2\pi f m^{-1}(x)} \frac{dx}{\left|\frac{\partial m}{\partial u}\right|} \quad where\; m^{-1}(x) = \frac{x}{a}; \quad \left|\frac{\partial m}{\partial u}\right| = |J| = a$$

$$h(au) \leftrightarrow \frac{1}{a} \int h(x) e^{-i2\pi \frac{fx}{a}} dx$$

$$h(au) \leftrightarrow \frac{1}{a} H\left(\frac{f}{a}\right)$$

Reciprocal Relationship



Magnification yields less high-freq
(more blurred)

Minification introduces higher freqs
(more visual detail)

Intuition: $f = 1 / T$
Consequences: narrow filters in spatial domain (desirable) yield wide frequency spectrums (undesirable). Tradeoff between accuracy and complexity.

Software (1)

```
resample1D(IN, OUT, INlen, OUTlen, filtertype, offset)
unsigned char *IN, *OUT;
int INlen, OUTlen, filtertype, offset;
{
  int i;
  int left, right;      // kernel extent in input
  int pixel;            // input pixel value
  double u, x;          // input (u) , output (x)
  double scale;         // resampling scale factor
  double (*filter)();   // pointer to filter fct
  double fwidth;        // filter width (support)
  double fscale;        // filter amplitude scale
  double weight;        // kernel weight
  double acc;           // convolution accumulator

  scale = (double) OUTlen / INlen;
```

Software (2)

```
switch(filtertype) {
case 0: filter = boxFilter;  // box filter (nearest nbr)
        fwidth = .5;
        break;
case 1: filter = triFilter;  // triangle filter (lin intrp)
        fwidth = 1;
        break;
case 2: filter = cubicConv;  // cubic convolution filter
        fwidth = 2;
        break;
case 3: filter = lanczos3;   // Lanczos3 windowed sinc fct
        fwidth = 3;
        break;
case 4: filter = hann4;      // Hann windowed sinc function        fwidth = 4;
        // 8-point kernel
        break;
}
```

Software (3)

```
if(scale < 1.0) {     // minification: h(x) -> h(x*scale)*scale
      fwidth = fwidth / scale;        // broaden  filter
      fscale = scale;                 // lower amplitude

      /* roundoff fwidth to int to avoid intensity modulation */
      if(filtertype == 0) {
              fwidth = CEILING(fwidth);
              fscale = 1.0 / (2*fwidth);
      }
} else fscale = 1.0;

// project each output pixel to input, center kernel,and convolve
for(x=0; x<OUTlen; x++) {
      /* map output x to input u: inverse mapping */
      u = x / scale;

      /* left and right extent of kernel centered at u */
      if(u - fwidth < 0) {
              left = FLOOR  (u - fwidth);
      else    left = CEILING(u - fwidth);
      right = FLOOR(u + fwidth);
```

Software (4)

```
                /* reset acc for collecting convolution products */
                acc = 0;

                /* weigh input pixels around u with kernel */
                for(i=left; i <= right; i++) {
                        pixel  = IN[ CLAMP(i, 0, INlen-1)*offset];
                        weight = (*filter)((u - i) * fscale);
                        acc    += (pixel * weight);
                }

                /* assign weighted accumulator to OUT */
                OUT[x*offset] = acc * fscale;
        }
}
```

Software (5)

```
double boxFilter(double t)
{
 if((t > -.5) && (t <= .5)) return(1.0);
 return(0.0);
}

double triFilter(double t)
{
 if(t < 0) t = -t;
 if(t < 1.0) return(1.0 - t);
 return(0.0);
}
```

Software (6)

```
double cubicConv(double t)
{
 double A, t2, t3;

 if(t < 0) t = -t;
 t2 = t  * t;
 t3 = t2 * t;

 A = -1.0;  // user-specified free parameter
 if(t < 1.0) return((A+2)*t3 - (A+3)*t2 + 1);
 if(t < 2.0) return(A*(t3 - 5*t2 + 8*t - 4));
 return(0.0);
}
```

Software (7)

```
double sinc(double t)
{
 t *= PI;
 if(t != 0) return(sin(t) / t);
 return(1.0);
}

double lanczos3(double t)
{
 if(t < 0) t = -t;
 if(t < 3.0) return(sinc(t) * sinc(t/3.0));
 return(0.0);
}
```

Software (8)

```
double hann4(double t)
{
 int N = 4;                // fixed filter width
 if(t < 0) t = -t;
 if(t < N)
     return(sinc(t) * (.5 + .5*cos(PI*t / N)));
 return(0.0);
}
```