CSC212
Data Structure

COMPUTER SCIENCE
CITY COLLEGE OF NEW YORK

Lecture 17

Trees, Logs and Time Analysis

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Topics

- Big-O Notation
- Worse Case Times for Tree Operations
- Time Analysis for BSTs
- Time Analysis for Heaps
- Logarithms and Logarithmic Algorithms

Big-O Notation

- The order of an algorithm generally is more important than the speed of the processor

Input size: n	O(log n)	O (n)	O (n^2)
# of stairs: n	$[\log_{10}n]+1$	3n	n^2+2n
10	2	30	120
100	3	300	10,200
1000	4	3000	1,002,000

Worst-Case Times for Tree Operations

- The worst-case time complexity for the following are all O(d), where d = the depth of the tree:
  - Adding an entry in a BST, a heap or a B-tree;
  - Deleting an entry from a BST, a heap or a B-tree;
  - Searching for a specified entry in a BST or a B-tree.

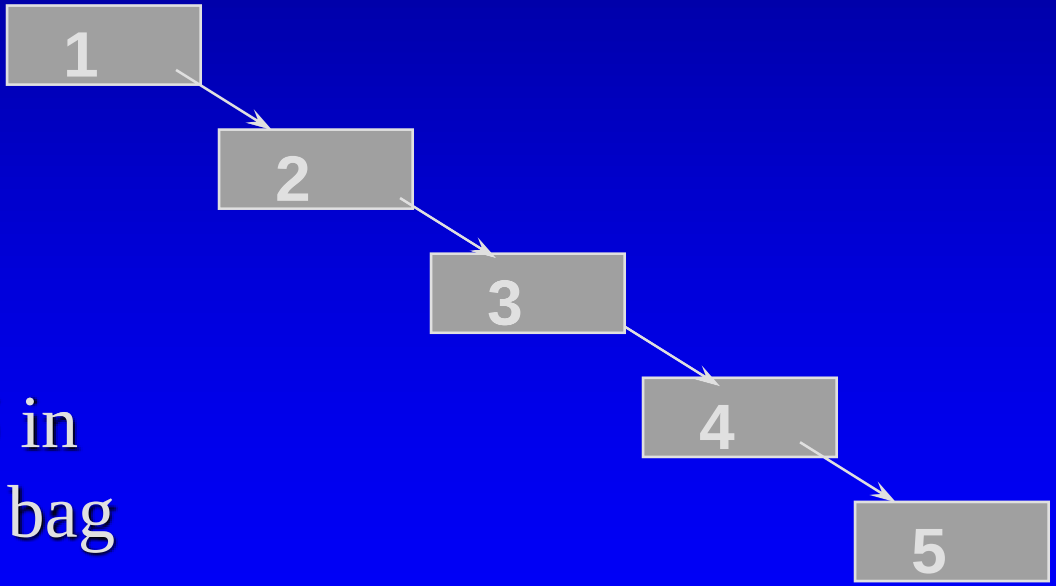
- This seems to be the end of our Big-O story...but

What's d, then?

- Time Analyses for these operations are more useful if they are given in term of the number of entries (n) instead of the tree's depth (d)
- Question:
  - What is the maximum depth for a tree with n entries?

Time Analysis for BSTs

- Maximum depth of a BST with n entries: n-1



- An Example:

  Insert 1, 2, 3,4,5 in that order into a bag using a BST

Worst-Case Times for BSTs

- Adding, deleting or searching for an entry in a BST with n entries is O(d), where d is the depth of the BST
- Since d is no more than n-1, the operations in the worst case is (n-1).
- Conclusion: the worst case time for the add, delete or search operation of a BST is O(n)

Time Analysis for Heaps

- A heap is a complete tree
- The minimum number of nodes needed for a heap to reach depth d is 2^d :
  - $= (1 + 2 + 4 + \ldots + 2^{d-1}) + 1$
  - The extra one at the end is required since there must be at least one entry in level d
- Question: how to add up the formula?

Time Analysis for Heaps

- A heap is a complete tree
- The minimum number of nodes needed for a heap to reach depth d is 2^d :
- The number of nodes n >= 2^d
- Use base 2 logarithms on both side
  - $\log_2 n >= \log_2 2^d = d$
  - Conclusion: $d <= \log_2 n$

Worst-Case Times for Heap Operations

- Adding or deleting an entry in a heap with n entries is O(d), where d is the depth of the tree
- Because d is no more than $\log_2 n$, we conclude that the operations are O(log n)
- Why we can omit the subscript 2 ?

Logarithms (log)

- Base 10: the number of digits in n is $[\log_{10} n]+1$
  - $10^0 = 1$, so that $\log_{10} 1 = 0$
  - $10^1 = 10$, so that $\log_{10} 10 = 1$
  - $10^{1.5} = 32+$, so that $\log_{10} 32 = 1.5$
  - $10^3 = 1000$, so that $\log_{10} 1000 = 3$
- Base 2:
  - $2^0 = 1$, so that $\log_2 1 = 0$
  - $2^1 = 2$, so that $\log_2 2 = 1$
  - $2^3 = 8$, so that $\log_2 8 = 3$
  - $2^5 = 32$, so that $\log_2 32 = 5$
  - $2^{10} = 1024$, so that $\log_2 1024 = 10$

Logarithms (log)

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  - $10^{1.5} = 32+$, so that $\log_{10} 32 = 1.5$
  - $10^3 = 1000$, so that $\log_{10} 1000 = 3$
- Base 2:
  - $2^3 = 8$, so that $\log_2 8 = 3$
  - $2^5 = 32$, so that $\log_2 32 = 5$
- Relation: For any two bases, a and b, and a positive number n, we have
  - $\mathbf{\log_b n = (\log_b a) \log_a n} = \log_b a^{(\log_a n)}$
  - $\mathbf{\log_2 n} = (\log_2 10) \log_{10} n = (5/1.5) \log_{10} n = \mathbf{3.3 \log_{10} n}$

Logarithmic Algorithms

- Logarithmic algorithms are those with worst-case time O(log n), such as adding to and deleting from a heap
- For a logarithm algorithm, doubling the input size (n) will make the time increase by a fixed number of new operations
- Comparison of linear and logarithmic algorithms
  - n= m = 1 hour -> $\log_2 m \approx 6$ minutes
  - n=2m = 2 hour -> $\log_2 m + 1 \approx 7$ minutes
  - n=8m = 1 work day -> $\log_2 m + 3 \approx 9$ minutes
  - n=24m = 1 day&night -> $\log_2 m + 4.5 \approx 10.5$ minutes

Summary

- Big-O Notation :
  - Order of an algorithm versus input size (n)
- Worse Case Times for Tree Operations
  - O(d), d = depth of the tree
- Time Analysis for BSTs
  - worst case: O(n)
- Time Analysis for Heaps
  - worst case O(log n)
- Logarithms and Logarithmic Algorithms
  - doubling the input only makes time increase a fixed number