Line Segment Visibility: Seeing all But Some Arbitrarily Small Amount

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Combinatorial Bounds - The Exact Case

We are interested in studying a family of line segment visibility problems, related to classical art gallery problems, which are motivated by monitoring requirements in commercial data centers. Given a collection of non-overlapping line segments in the interior of a rectangle, and a requirement to monitor the segments from one side or the other, we examine the problem of finding a minimal guard set. We are interested in combinatorial bounds of problem variants where the problem solver gets to decide which side of the segments to guard, the problem poser gets to decide which side to guard, as well as the case where both sides of all segments must be guarded.

From an applications point of view, the poser's choice problem corresponds to the case where we have a set of server racks, with equipment facing in one direction or the other per rack and we have to place thermal imaging cameras to see all the relevant air intakes, to make sure this air is not too warm. The solver's choice variant corresponds to the case where the rack locations are given but we not only can choose where to place cameras but also how to orient the air intake sides of the servers so that we can utilize as few thermal imaging cameras as possible. In the variant of the problem where we must see both sides of all racks, each rack contains servers, some of whose air intakes face in one direction and some of whose air intakes face in the other direction.

By a combinatorial bound for a given problem we mean, given N segments, in the worst case, how many guards are needed to see each of the N segments from the requisite side(s)?

The following tables show the state of our knowledge regarding both upper and lower bounds for each of the problems we have examined, up to constant factors. Along the vertical (left) edge of the table, we have listed the different types of orientations we have considered and along the horizontal (top) edge of the table we have listed the sidedness constraints we have considered.

Sidedness Constraint / Segment Orientation	Solver's Choice	Solver's Choice	Poser's Choice (All segments from same specified side)	Poser's Choice	Both Sides
Vertical	U n/3 L n/3	U n/3 L n/3	U n/2 L n/2	U n/2 L n/2	U 2n/3 L 2n/3
Orthogonal	U n/2 L n/3	U n/2 L 3n/8	U n/2 L n/2	U 2n/3 L 2n/3	U 2n/3 L 2n/3
Arbitrary	U n/2 L 2n/5	U 3n/4 L 2n/5	U 3n/4 L n/2	U 3n/4 L 2n/3	U 4n/5 L 4n/5

Figure 1. A table summarizing what we know about the combinatorial bounds for the various problem variants. U denotes the upper bound, L the lower bound. All stated results are modulo constant factors.

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In the first column, the solver chooses which side to see, but can choose to see some points of the line segment from one side and some points from the other side. In the second column the solver must see all segments entirely from one side or the other. In the third column, the poser dictates which side of each segment must be seen, but is constrained to dictate that segments be seen from a consistent vantage-point. Thus if all segments are axis-aligned, then the constraint might be to see all vertical segments from the right and all horizontal segments from the bottom. If segments are arbitrarily aligned, to prescribe a consistent sidedness we demand that the problem poser indicate, for an arbitrary 180 degree rotation of a line through the origin, a consistent "top" side of that line, in other words a sidedness that does not change as the line sweeps out its rotation. This sidedness can conveniently be prescribed by placing a rotational symbol to one side or the other of the stating ray, as in Figure 2.

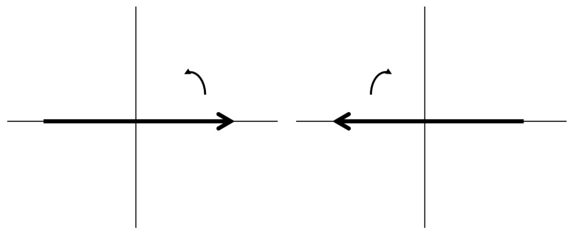

Figure 2. Specifying a consistent sidedness for line segments in the plane: in the figure on the left, as the ray sweeps out its 180 degree rotation, horizontal segments must be seen from the top (indicated by the arc-sweeping symbol being placed above the starting ray), and then vertical segments must be seen from the left. On the other hand, in the orientation indicated in the right hand figure, horizontal segments again must be seen from above, but vertical segments must be seen from the right.

Combinatorial Bounds - The All But Delta Case

In examples of the Poser's choice variant of the problem we typically indicate which side of each segment must be seen by introducing little "tick" marks. For example in Figure 3, due to Csaba, the sides we must guard are

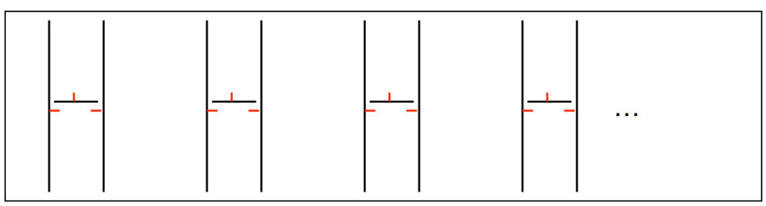

Figure 3. An example of N axis aligned segments for the Poser's Choice variant of the problem. The red "tick" marks indicate which side of each segment must be seen. In this case it is easy to see that $\frac{2N}{3}$ guards are required to see all segments entirely from the specified sides.

given by the red "tick" marks. Note that if we try to place a single camera to see all of the specified sides of any one of the "H"s, the camera necessarily leaves a small blind spot on one of the vertical segments. In order to pick up the blind spot, a second camera must be placed very close to the "H" and therefore is of no help in seeing anything in any other "H". Thus, given N such axis-aligned segments, we must use $\frac{2N}{3}$ cameras to see everything. This is where the lower bound of $\frac{2N}{3}$ comes from in Figure 4 .in the second Poser's choice column for the axis-aligned case.

However, in any real problem we don't really care about having an arbitrarily small blind spot. Thus we are led to consider the case of the above mentioned problems where we allow ourselves to miss an arbitrarily small amount along all segments (and hence an arbitrarily small amount in total). Eli and Jon introduced this family of problems in a recently submitted paper, and provided the following table of relatively easy to obtain bounds that we believe can be readily sharpened.

Sidedness Constraint / Segment Orientation	Solver's Choice	Solver's Choice	Poser's Choice (All segments from same specified side)	Poser's Choice	Both Sides
Vertical	U n/3 L n/3	U n/3 L n/3	U n/2 L n/2	U n/2 L n/2	U 2n/3 L 2n/3
Orthogonal	U n/2 L n/3	U n/2 L n/3	U n/2 L n/2	U n/2 L n/2	U 4n/5 L 2n/3
Arbitrary	U n/2 L 2n/5	U 3n/4 L 2n/5	U 3n/4 L n/2	U 3n/4 L n/2	U 4n/5 L 4n/5

Figure 4. A table summarizing what we know about the combinatorial bounds for the various problem variants. U denotes the upper bound, L the lower bound. All stated results are modulo constant factors.

1 Results

This is where we stood at the beginning of the workshop. We obtained a number of new results, and stated a couple of new problems, during the workshop – the details of which will go here, with hopefully additional results to follow.