Packing and Covering a Polygon with Geodesic Disks

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1. Motivation and Related Work

Packing and covering problems are among the most studied problems in discrete geometry. Nevertheless, most of the literature focus on packings and coverings using Euclidean balls, which is a somewhat unrealistic assumption for practical problems. A prominent practical example is the *Facility Location* (*k-Center*) problem (see for example [3]) in buildings or other constrained areas. In this a setting the relevant distance metric is the shortest path metric and not the Euclidean distance. Such a problem occurs when a mobile robot is navigating in a room such as a data center (see [8] and [9] for such an example), which is naturally modeled as a polygon, and we are interested in placing charging stations in such a way that the worst case travel time of the robot to the closest station gets minimized [7]. The shortest path distance is also referred to as the *geodesic distance* and, for two points u and v in a polygon P, it is denoted by $d(u, v)$ and defined as the length of the shortest path between u and v which stays inside P. Furthermore, we define a closed *geodesic disk* D of radius r centered at a point $v \in P$, as the set of all points in P whose geodesic distance to v is at most r.

In this paper we would like to initiate the studies of packing and covering problems in polygons using geodesic disks.

2. Results

Problem 1 (Geodesic Unit Disk Packing)**.** *Given a polygon, find a maximum cardinality packing with geodesic disks of radius* 1*.*

Theorem 2. *Geodesic Unit Disk Packing can be* 2*-approximated in time* $O(K(n+K)\log^2(n+K))$*, where* n *is the number of vertices of the polygon and* K *is the size of the output.*

Theorem 3. *Geodesic Unit Disk Packing is* NP*-hard in polygons with holes.*

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Problem 4 (Geodesic Unit Disk Covering)**.** *Given a polygon P, find a cover of P with fewest geodesic unit disks.*

Theorem 5. *Geodesic Unit Disk Covering is* NP*-hard in polygons with holes.*

Problem 6 (Geodesic k-Covering)**.** *Given a polygon P, possibly with holes, find a cover of P with k geodesic disks whose maximal radius is minimized.*

Theorem 7. *Geodesic k-Covering can be 2-approximated in time $O(k^2(n+k)\log(n+k))$.*

Problem 8 (Geodesic k-Packing)**.** *Given a polygon P, possibly with holes, pack k geodesic disks whose minimum radius is maximized.*

Theorem 9. *Geodesic k-Packing can be 4-approximated in time $O(k^2(n+k)\log(n+k))$.*

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