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Modifications due to local field corrections of the electromagnetically induced transparency propagation parameters in a driven optically dense three-level cascade system

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Abstract

We compute, under conditions of electromagnetically induced transparency, the frequency dependence of the transmission coefficient and the group velocity of a probe pulse resonant with the lower transition of a three-level cascade system in the presence of a cw pump field resonant with the system's upper transition. We show that the presence of local field corrections can substantially modify the profile of both these transmission-related quantities. © 2000 Published by Elsevier Science B.V.

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1. Introduction

The main characteristic of an optically dense medium is the short absorption length of light and the presence of local field corrections (LFC) to the Maxwell field, resulting, inter alia, in a shift in the absorption spectral profile [1–3]. However, due to the strong absorption of light in such medium, reflectivity measurement [4,5] was among the few experimental techniques available to probe these frequency shifts and other nonlinear dynamics induced by the Lorentz corrections. Furthermore, the dependence of these shifts on the population of the different levels could be inferred only indirectly. An interesting question then arises, namely: can we use electromagnetically induced transparency (EIT) [6,7] present in a driven cascade system [7–10] to probe the details of the spectral profile dynamics in a transmission experiment?

In this paper, we consider a cascade medium which is optically dense, i.e. the absorption length for a weak signal resonant with the lower transition and propagating in the gas system initially in the ground state, is much shorter than the corresponding radiation wavelength. In the presence of a pump field, resonant with the upper transition, the phenomenon of EIT acts catalytically on the probe field and leads to a suppression of its absorption coefficient near the lower transition

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resonance frequency while greatly decreasing the value of its group velocity. In effect, we show that this technique does not allow us only the means to measure the position of the maximum of the absorption curve, but also provide us with the means to measure the derivatives of the index of refraction in this neighborhood.

We solve numerically the coupled integro-differential Maxwell–Bloch equations describing the electromagnetic radiation propagation in the pressure broadened three-level cascade medium. We do not assume, in Maxwell equations, the slowly varying envelope approximation in space, and we make, in Bloch equations, the distinction between the Maxwell and Lorentz fields.

Our analysis does not include the effects of the gas collision with the walls of the container, which, if kept hot to prevent condensation, can be neglected in the considered dense gas regime.

2. Maxwell–Bloch equations

The dynamics of the interaction of the electromagnetic field with the pressure-broadened three-level atoms system is described by the coupled Maxwell–Bloch equations. Neglecting the counter-rotating term in the Hamiltonian, Bloch equations, including the local field corrections, in the presence of both the upper and the lower Maxwell fields, φ_{cb} and φ_{ba}, are given by [11]:

$$\frac{\partial \rho_{aa}}{\partial \tau} = \Gamma_{ba}\rho_{bb} - \mathrm{i}\varphi_{ba}\rho_{ba}^{*} + \mathrm{i}\varphi_{ba}^{*}\rho_{ba} \tag{1}$$

$$\begin{aligned}\frac{\partial \rho_{bb}}{\partial \tau} = {} & -\Gamma_{ba}\rho_{bb} + \Gamma_{cb}\rho_{cc} + \mathrm{i}\varphi_{ba}\rho_{ba}^{*} - \mathrm{i}\varphi_{ba}^{*}\rho_{ba} \\ & - \mathrm{i}\varphi_{cb}\rho_{cb}^{*} + \mathrm{i}\varphi_{cb}^{*}\rho_{cb}\end{aligned} \tag{2}$$

$$\frac{\partial \rho_{cc}}{\partial \tau} = -\Gamma_{cb}\rho_{cc} + \mathrm{i}\varphi_{cb}\rho_{cb}^{*} - \mathrm{i}\varphi_{cb}^{*}\rho_{cb} \tag{3}$$

$$\frac{\partial \rho_{cb}}{\partial \tau} = -(\gamma_{cb} + \mathrm{i}\Delta_{cb})\rho_{cb} - \mathrm{i}\varphi_{cb}^{\mathrm{L}}(\rho_{cc} - \rho_{bb}) - \mathrm{i}\varphi_{ba}^{\mathrm{L}^*}\rho_{ca} \tag{4}$$

$$\frac{\partial \rho_{ba}}{\partial \tau} = -(\gamma_{ba} + \mathrm{i}\Delta_{ba})\rho_{ba} - \mathrm{i}\varphi_{ba}^{\mathrm{L}}(\rho_{bb} - \rho_{aa}) + \mathrm{i}\varphi_{cb}^{\mathrm{L}^*}\rho_{ca} \tag{5}$$

$$\frac{\partial \rho_{ca}}{\partial \tau} = -(\gamma_{ca} + \mathrm{i}\Delta_{cb} + \mathrm{i}\Delta_{ba})\rho_{ca} - \mathrm{i}\varphi_{ba}^{\mathrm{L}}\rho_{cb} + \mathrm{i}\varphi_{cb}^{\mathrm{L}}\rho_{ba} \tag{6}$$

where τ is the time normalized to the classical Lorentz shift ω_{L}, $\tau = \omega_{\mathrm{L}} t$, $\omega_{\mathrm{L}} = Nd^2/6\hbar\varepsilon$, N is the atomic number density, d is the dipole transition matrix element for the upper and lower transitions assumed here for simplicity to be equal, the deltas corresponding to the normalized detuning of the fields frequencies from that of the upper and lower transition resonance frequencies ($\Delta_{cb} = (\omega_{cb} - \omega_{\mathrm{pump}})/\omega_{\mathrm{L}}$, $\Delta_{ba} = (\omega_{ba} - \omega_{\mathrm{probe}})/\omega_{\mathrm{L}}$), Γs are the normalized natural decay constants, which will be neglected here, as compared to the γs, the normalized transverse decay times, due to collisions, the L superscripted fields are the Lorentz fields, and the normalized fields are the Rabi frequencies of the corresponding fields normalized to ω_{L}, i.e. $\varphi_{cb} = dE_{cb}/\hbar\omega_{\mathrm{L}}$.

More specifically, and in order to study the present problem without the effects of additional complications due to the particular atomic structures details, we assume, here, the model where the atomic transition frequencies are separated by values much larger than the pressure broadened width, but infinitely smaller than the transition frequencies, and such that the spin structures of the uppermost and ground states are the same. Under such assumptions, we have:

$$\varphi_{cb}^{\mathrm{L}} = \varphi_{cb} + \alpha\rho_{cb} \tag{7}$$

$$\varphi_{ba}^{\mathrm{L}} = \varphi_{ba} + \alpha\rho_{ba} \tag{8}$$

$$\gamma_{cb} = a(\rho_{cc} + \rho_{bb}) \tag{9}$$

$$\gamma_{ba} = a(\rho_{bb} + \rho_{aa}) \tag{10}$$

$$\gamma_{ca} = 0 \tag{11}$$

Neglecting the effects of quantum modifications to the LFC, the constant α is equal to one and the constant a, for example, for a $J = 0 \rightarrow J = 1$ transition is equal, as was shown in Refs. [1,2], to 1.7293. We shall use for illustrative purposes, this value of a in subsequent calculations, and vary the value of α, the measure of the Lorentz effects strength, to investigate the effects of LFC on EIT.

Maxwell equations for the fields are given by:

$$\frac{\partial^2 \varphi_{cb}}{\partial \bar{z}^2} - \frac{1}{V^2}\frac{\partial^2 \varphi_{cb}}{\partial \tau^2} = -3\rho_{cb} \tag{12}$$

$$\frac{\partial^2 \varphi_{ba}}{\partial \bar{z}^2} - \frac{1}{V^2}\frac{\partial^2 \varphi_{ba}}{\partial \tau^2} = -3\rho_{ba} \tag{13}$$

where $\bar{z} = k_0 z$, and k_0 is the wave number corresponding to the atomic transitions, taken here for simplicity to be the same for both the upper and lower transitions, and where V is the velocity of light in vacuum, equal in the present units to $V = (\omega_0/\omega_{\mathrm{L}}) \equiv \Omega_0$.

Using the Green's function for the Helmholtz equation, the above differential forms of Maxwell equations can be also written in an integral form [12–14]:

$$\begin{aligned}\varphi_{cb}(\bar{z},\tau) &= \varphi_{cb}^{\mathrm{in}}(\bar{z}=0,\tau)\exp(\mathrm{i}\bar{z}) \\ &\quad + \mathrm{i}\frac{3}{2}\int_0^{\bar{L}} \mathrm{d}\bar{z}' \exp(\mathrm{i}|\bar{z}-\bar{z}'|) \\ &\quad \times \rho_{cb}\left(\bar{z}', \tau - \frac{|\bar{z}-\bar{z}'|}{V}\right)\end{aligned} \tag{14}$$

$$\begin{aligned}\varphi_{ba}(\bar{z},\tau) &= \varphi_{ba}^{\mathrm{in}}(\bar{z}=0,\tau)\exp(\mathrm{i}\bar{z}) \\ &\quad + \mathrm{i}\frac{3}{2}\int_0^{\bar{L}} \mathrm{d}\bar{z}' \exp(\mathrm{i}|\bar{z}-\bar{z}'|) \\ &\quad \times \rho_{ba}\left(\bar{z}', \tau - \frac{|\bar{z}-\bar{z}'|}{V}\right)\end{aligned} \tag{15}$$

where the incoming fields are the external fields to the present atomic-field system. We shall use the above integral form of Maxwell wave equation in subsequent numerical calculations.

3. Probe field dispersion relation in the absence of local field corrections

In this section, we compute, using the dispersion relation for the probe field (i.e. the relation that relates its wave vector with its frequency), approximate expressions for the group velocity and the absorption coefficient of the probe signal propagating in a medium consisting of driven three-level cascade-structure atoms, neglecting the LFC. This will help us determine the choice of parameters that we will utilize in the numerical solutions of Section 4, where we will solve the complete Maxwell–Bloch system of equations, including LFC.

In the absence of LFC, and under the assumption that $|\varphi_{ba}| \ll |\varphi_{cb}|$, the off-diagonal matrix element ρ_{ba}, the source term for the probe field, is approximately linear in the field φ_{ba}, and is given by:

$$\rho_{ba} = \frac{\mathrm{i}\varphi_{ba}}{\left(\gamma_{ba} + \mathrm{i}\Delta_{ba} + \frac{\varphi_{cb}^*\varphi_{cb}}{\gamma_{ca}+\mathrm{i}(\Delta_{cb}+\Delta_{ba})}\right)} \tag{16}$$

Combining Eqs. (13) and (16), we can write the following dispersion relation:

$$\begin{aligned}K^2 &= \frac{\Omega^2}{\Omega_0^2}\left[1 + \frac{\Omega_0^2}{\Omega^2}\frac{3\mathrm{i}}{\left(\gamma_{ba} + \mathrm{i}\Delta_{ba} + \frac{\varphi_{cb}^*\varphi_{cb}}{\gamma_{ca}+\mathrm{i}(\Delta_{cb}+\Delta_{ba})}\right)}\right] \\ &= \frac{\Omega^2}{\Omega_0^2} n^2(\Omega)\end{aligned} \tag{17}$$

The real and imaginary part of the index of refraction $n(\Omega)$ give respectively the linear dispersion and the absorption coefficients. The group velocity is given by:

$$v_{\mathrm{g}} = \frac{\Omega_0}{\mathrm{Re}(m(\Omega))} \tag{18}$$

where

$$\begin{aligned}m(\Omega) &= n(\Omega) + \Omega\frac{\partial n(\Omega)}{\partial \Omega} \\ &= n(\Delta_{ba}) - (\Omega_0 - \Delta_{ba})\frac{\partial n(\Delta_{ba})}{\partial \Delta_{ba}}\end{aligned} \tag{19}$$

In Fig. 1, we plot respectively the imaginary and real parts of $n(\Delta_{ba})$ and $m(\Delta_{ba})$ as function of Δ_{ba}, for a moderate value of the φ_{cb} field. Noting that the spectral width of these quantities is of the order of $\omega_{\mathrm{L}}/1000$ (in ordinary units), it is then possible to have a probe pulse propagate in this medium over few wavelength with little absorption and with a group velocity a million time smaller than the velocity of light in vacuum. The delay in the pulse envelope in the present normalized units will be given by:

$$\tau_{\mathrm{d}} = \omega_{\mathrm{L}}\frac{L}{c}\mathrm{Re}(m) \tag{20}$$

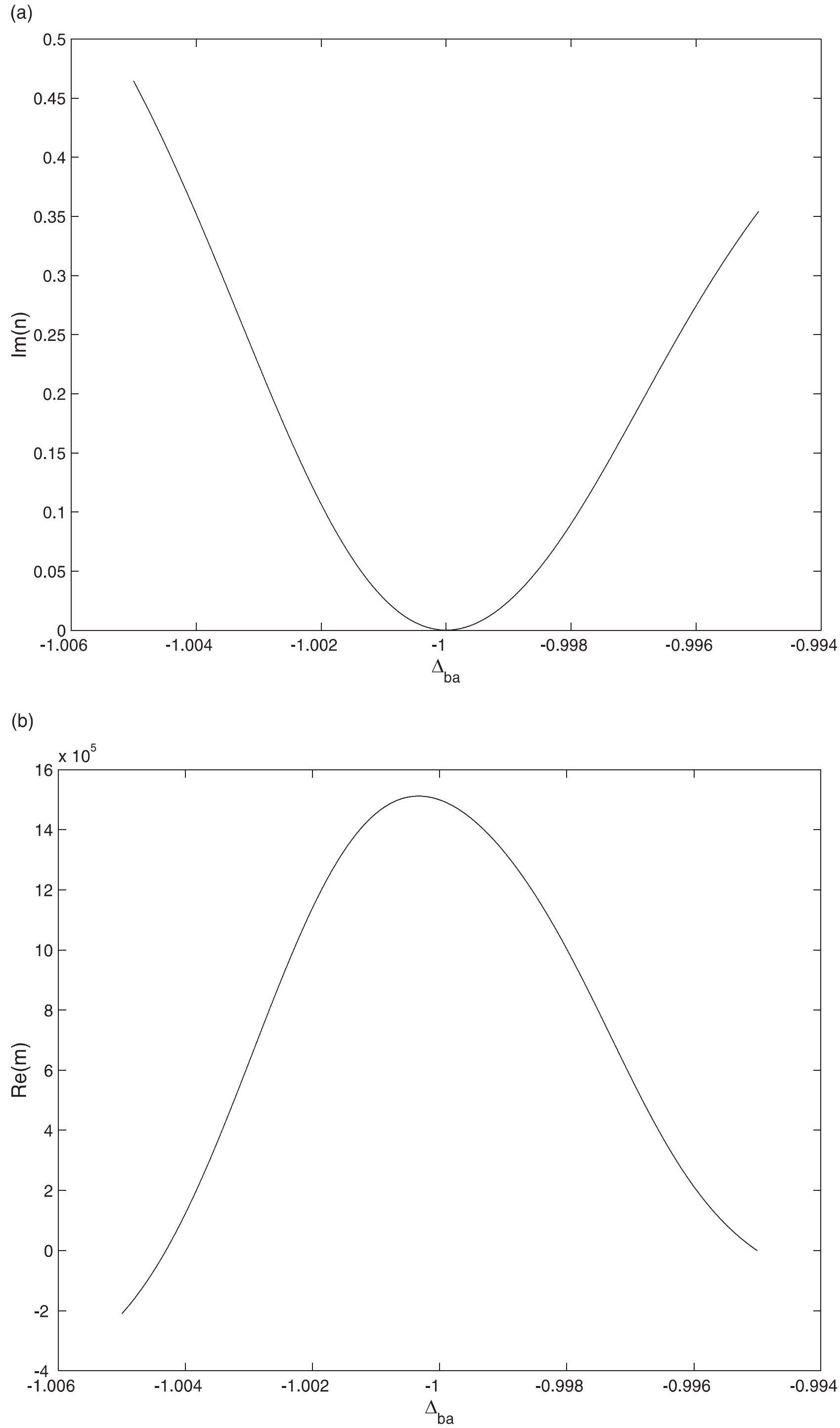

Fig. 1. The absorption coefficient and the group velocity reduction factor are plotted as function of the probe detuning in a driven optically dense cascade ladder three-level atom system. $\Delta_{cb} = 1$, $\Omega_0 = 10^4$, $\varphi_{cb}^{\mathrm{in}} = 0.1$, $\alpha = 0$.

The above estimates for the weak pulse propagation coefficients in the driven medium were deduced through an analysis of the dispersion relation. As we pointed earlier, we shall consider, in Section 4, the exact propagation problem with the same parameters as those of the above figure and study the modifications to the transmission coefficients of interest as function of the magnitude of the LFC. But before we go into that, and to gain an insight into the orders of magnitude of the quantities that we are investigating, it is worthwhile computing the propagation parameters, for the case of even smaller values of the φ_{cb} pump field as that considered in Fig. 1.

In Fig. 2, we plot respectively the real and imaginary parts of $n(\Delta_{ba})$ and $m(\Delta_{ba})$ as function of Δ_{ba}, for a small value of the φ_{cb} field. We note that, in this case, the group velocity of light can be slowed down by another factor of 100 than for the previous parameters, however this reduction in the group velocity can only be achieved for an incoming probe pulse whose spectral width is 100 times smaller than that associated with the previous figure, i.e. having a duration 100 times longer than in the previous figure. This trade-off between a reduction in the group velocity of the probe and a reduction in the probe's permissible spectral width is a general feature common to many instances of systems with EIT features.

4. Dependence of the transmission coefficients on the magnitude of the local field correction

Next, we solve numerically, with no simplifications, the coupled Maxwell–Bloch equations for the parameters of Fig. 1, through an algorithm similar to that previously detailed [14]. Our goal is to obtain directly, through direct integration of the system of equations, the shape and the position of a probe propagating, under the general condition of EIT. In particular, we desire to investigate the stability of EIT in the presence of LFC and the modifications that these corrections bring to the coefficients of the probe's propagation.

We assume here, that the pump field is a cw field, and chose the incoming probe pulse parameters such that the pulse's spectral width falls within the low absorption coefficient window, as estimated in Fig. 1. In Fig. 3, we summarize the results of the propagation calculations for different values of the parameter α.

We note, in particular that for $\alpha = 0$, i.e. the case that the LFC are neglected, the exact results differ, at exact resonance, by less than 5% from those obtained from those estimated in the previous section, using uniquely the imaginary part of $n(\Delta_{ba})$ and the real part of $m(\Delta_{ba})$. The small deviation between the two sets of results is due to the fact that we are neglecting the dispersion and chirping effects induced by the real part of $n(\Delta_{ba})$ and the imaginary part of $m(\Delta_{ba})$, which have the effect of modifying the pulse shape. These effects are small here due to the important fact that both neglected quantities have odd symmetry near the spectral maximum and thus tend to modify only slightly the pulse shape.

The other values of α corresponding to the other traces in the figure, i.e. 1 and 4, correspond respectively to those values obtained for the classically determined (i.e. no quantum corrections) frequency shift [1,2], and the enhanced but below the LFC induced intrinsic bistability value [3,15].

As can be deduced from the above figures, the present technique for measuring LFC, based on EIT, gives a sharper determination for the position of the maximum of the absorption coefficient than through a reflectivity experiment because the transmission coefficient dependence on the absorption coefficient is exponential (Beer's law) while in a reflectivity experiment, the dependence is almost linear.

Furthermore, note that an examination of Fig. 3 reveals that, the dependence of the group velocity on α can be substantial which suggests a promising novel experimental technique for investigating LFC, supplementary to the usual direct spectral technique.

5. Conclusion

In this paper, using the EIT effect, we investigated the modifications to the transmission coefficient and to the group velocity of a probe propagating in a driven ladder cascade three-level atom due to the

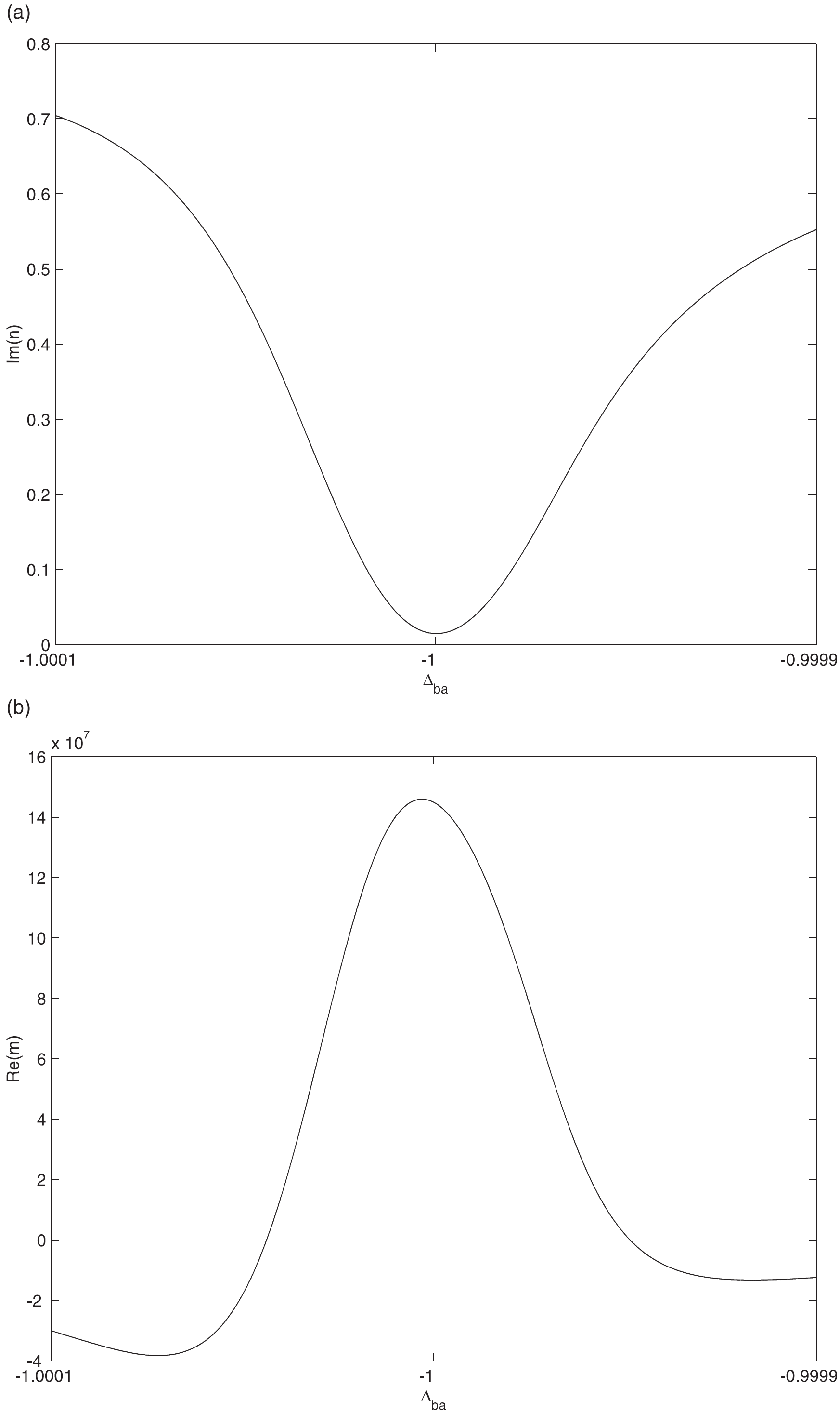

Fig. 2. Same as Fig. 1, but for a smaller driving field: $\Delta_{cb} = 1$, $\Omega_0 = 10^4$, $\varphi_{cb}^{\rm in} = 0.1$, $\alpha = 0$.

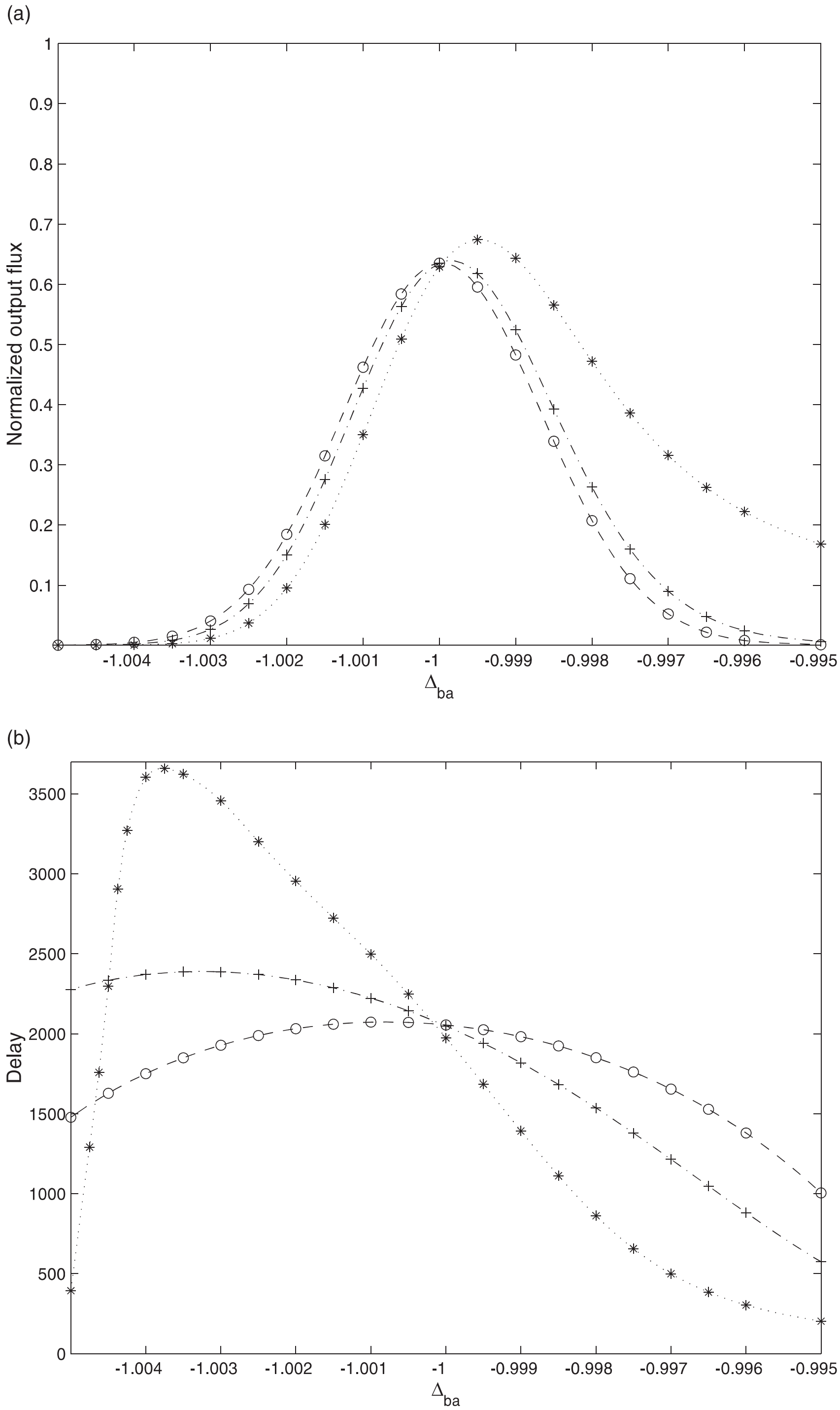

Fig. 3. The φ_{ba} probe field transmission coefficient and the pulse group delay for a driven dense ladder system having the following parameters: $\Delta_{cb} = 1$, $\Omega_0 = 10^4$, $\varphi_{cb}^{\rm in} = 0.1$, $L = 2.25\lambda$. The pump field is a cw field while the in coming probe field envelope is: $\varphi_{ba}^{\rm in}(\bar{z} = 0, \tau) = (0.001)\exp(-(\tau - 3000)^2/10^6)$ $\circ$: $\alpha = 0$; $+$: $\alpha = 1$; $*$: $\alpha = 4$.

presence of LFC. Our results suggest a possible new experimental technique for investigating LFC. Furthermore, the computational simulations show clearly that the EIT effect is stable in a dense gas medium, in spite of LFC, i.e. the collisional dephasing does not destroy the coherence effects responsible for the establishment of EIT.

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