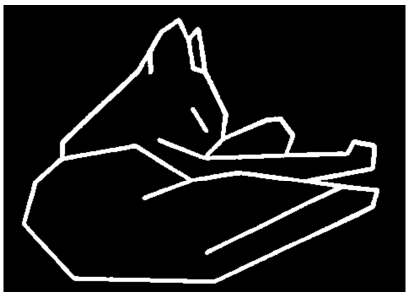
3D Computer Vision
and Video Computing

Introduction

CSc I6716
Spring 2011

Part I
Feature Extraction (2)



Edge Detection

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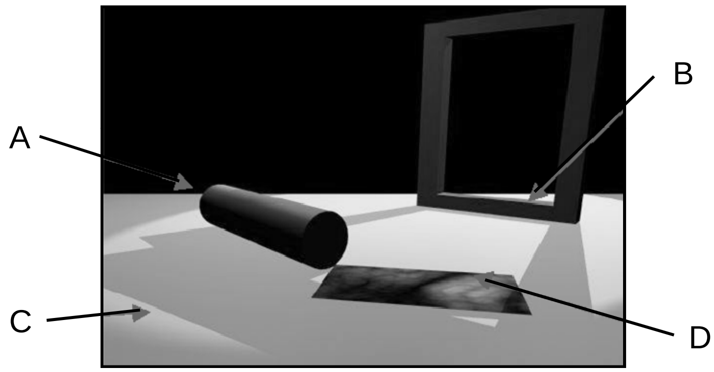
Edge Detection

- What's an edge?
  - "He was sitting on the Edge of his seat."
  - "She paints with a hard Edge."
  - "I almost ran off the Edge of the road."
  - "She was standing by the Edge of the woods."
  - "Film negatives should only be handled by their Edges."
  - "We are on the Edge of tomorrow."
  - "He likes to live life on the Edge."
  - "She is feeling rather Edgy."
- The definition of Edge is not always clear.
- In Computer Vision, Edge is usually related to a discontinuity within a local set of pixels.

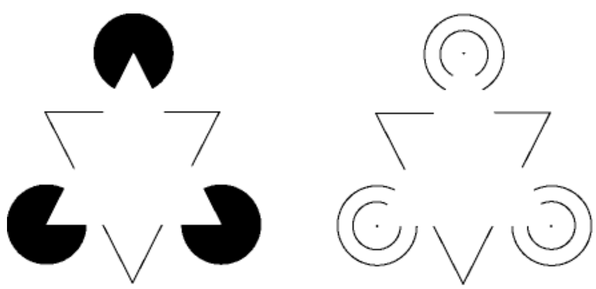
Discontinuities



- A: Depth discontinuity: abrupt depth change in the world
- B: Surface normal discontinuity: change in surface orientation
- C: Illumination discontinuity: shadows, lighting changes
- D: Reflectance discontinuity: surface properties, markings

Illusory Edges

Kanizsa
Triangles



- Illusory edges will not be detectable by the algorithms that we will discuss
- No change in image irradiance - no image processing algorithm can directly address these situations
- Computer vision can deal with these sorts of things by drawing on information external to the image (perceptual grouping techniques)

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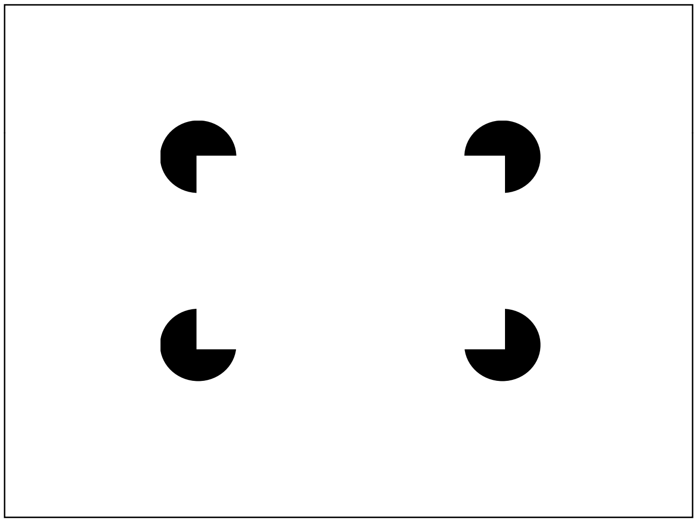
Another One



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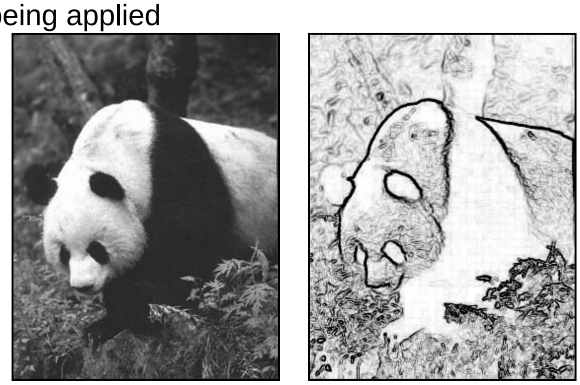
Goal

- Devise computational algorithms for the extraction of significant edges from the image.
- What is meant by significant is unclear.
  - Partly defined by the context in which the edge detector is being applied



Edgels

- Define a local edge or *edgel* to be a rapid change in the image function over a small area
  - implies that edgels should be detectable over a local neighborhood
- Edgels are NOT contours, boundaries, or lines
  - edgels may lend support to the existence of those structures
  - these structures are typically constructed from edgels
- Edgels have properties
  - Orientation
  - Magnitude
  - Position

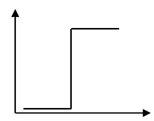
Outline

- First order edge detectors (lecture - required)
  - Mathematics
  - 1x2, Roberts, Sobel, Prewitt
- Canny edge detector (after-class reading)
- Second order edge detector (after-class reading)
  - Laplacian, LOG / DOG
- Hough Transform – detect by voting
  - Lines
  - Circles
  - Other shapes

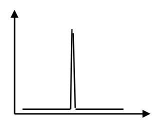
Locating Edgels

Rapid change in image => high local gradient => differentiation

f(x) = step edge

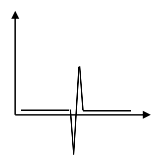


1^{st} Derivative f '(x)



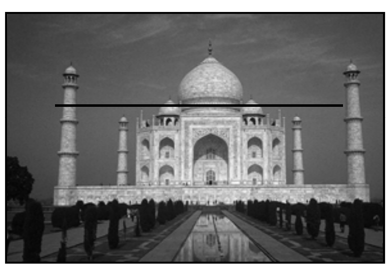
maximum

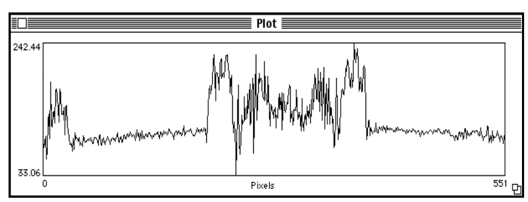
2^{nd} Derivative -f ''(x)

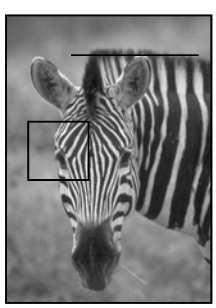


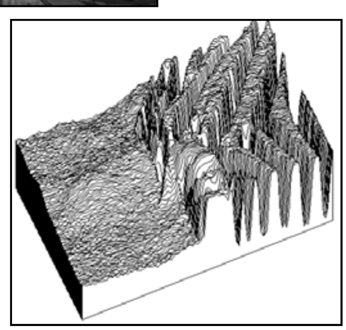
zero crossing

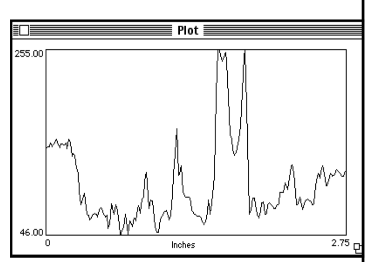
Reality



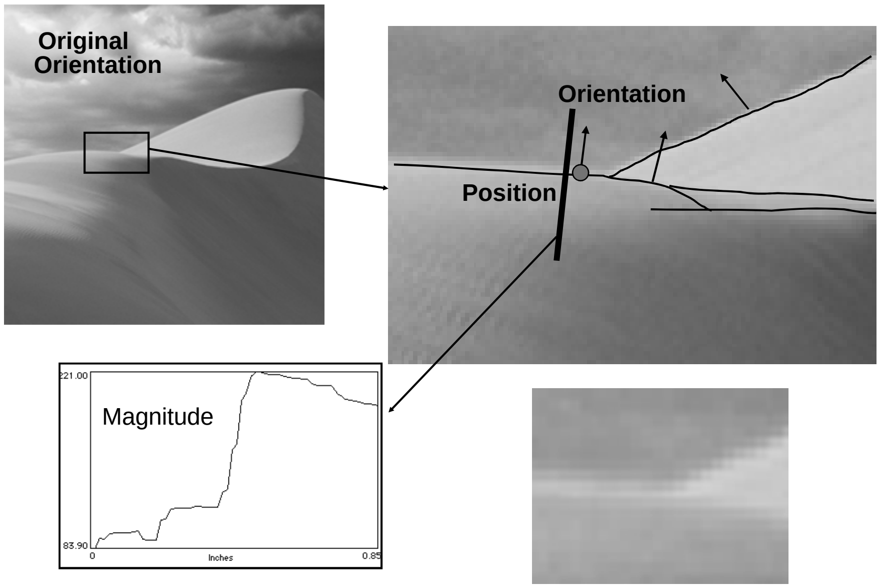








Properties of an Edge



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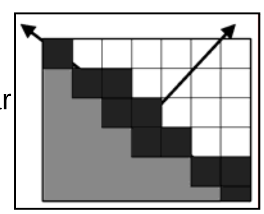
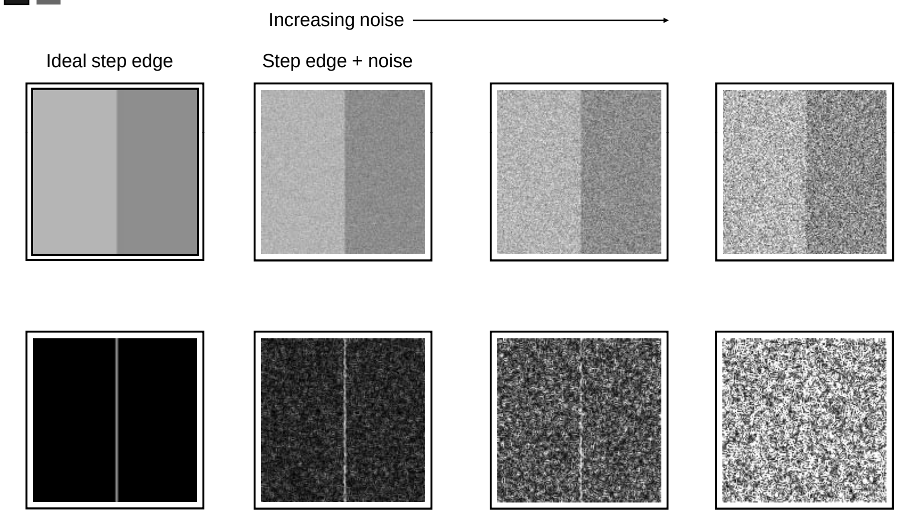
Quantitative Edge Descriptors

- Edge Orientation
  - Edge Normal - unit vector in the direction of maximum intensity change (maximum intensity gradient)
  - Edge Direction - unit vector perpendicular the edge normal
- Edge Position or Center
  - image position at which edge is located (usually saved as binary image)
- Edge Strength / Magnitude
  - related to local contrast or gradient - how rapid is the intensity variation across the edge along the edge normal.



Edge Degradation in Noise



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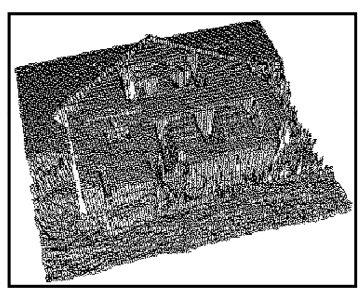
Real Image



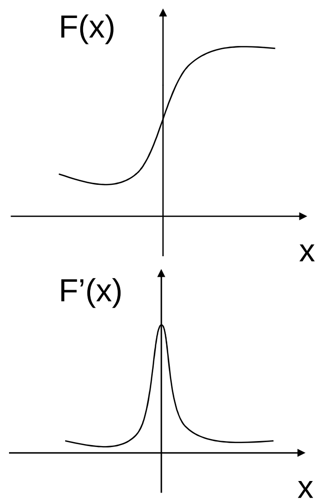
Edge Detection: Typical

- Noise Smoothing
  - Suppress as much noise as possible while retaining 'true' edges
  - In the absence of other information, assume 'white' noise with a Gaussian distribution
- Edge Enhancement
  - Design a filter that responds to edges; filter output high are edge pixels and low elsewhere
- Edge Localization
  - Determine which edge pixels should be discarded as noise and which should be retained
    - thin wide edges to 1-pixel width (nonmaximum suppression)
    - establish minimum value to declare a local maximum from edge filter to be an edge (thresholding)

Edge Detection Methods

- 1st Derivative Estimate
  - Gradient edge detection
  - Compass edge detection
  - Canny edge detector (*)
- 2nd Derivative Estimate
  - Laplacian
  - Difference of Gaussians
- Parametric Edge Models (*)

Gradient Methods



Edge= sharp variation

↓

Large first derivative

Gradient of a Function

- Assume f is a continuous function in (x,y). Then

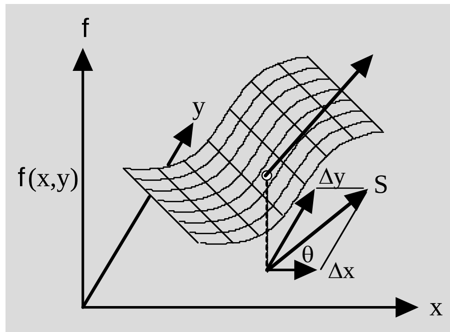
$$\Delta_x = \frac{\partial f}{\partial x}, \quad \Delta_y = \frac{\partial f}{\partial y}$$

- are the rates of change of the function f in the x and y directions, respectively.
- The vector (Δ_x, Δ_y) is called the gradient of f.
- This vector has a magnitude: $s = \sqrt{\Delta_x^2 + \Delta_y^2}$

  and an orientation: $\theta = \tan^{-1} \left(\frac{\Delta_y}{\Delta_x} \right)$

- θ is the direction of the maximum change in f.
- S is the size of that change.

Geometric Interpretation



- But
  - I(i,j) is not a continuous function.
- Therefore
  - look for discrete approximations to the gradient.

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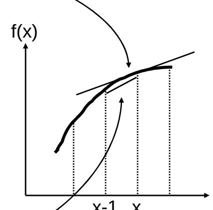
Discrete Approximations

$$\frac{df(x)}{dx} = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$\frac{df(x)}{dx} \cong \frac{f(x) - f(x-1)}{1}$$

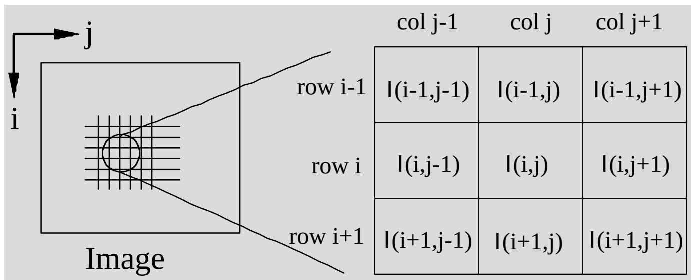


Convolve with

-1	1

In Two Dimensions

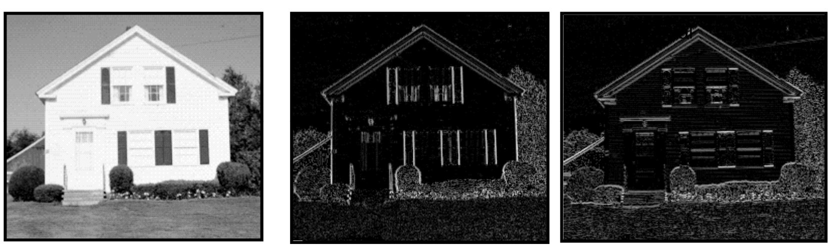
- Discrete image function I



- Derivatives ⇨ Differences

$\Delta_j I = \begin{bmatrix} -1 & 1 \end{bmatrix}$ $\qquad$ $\Delta_i I = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

1x2 Example



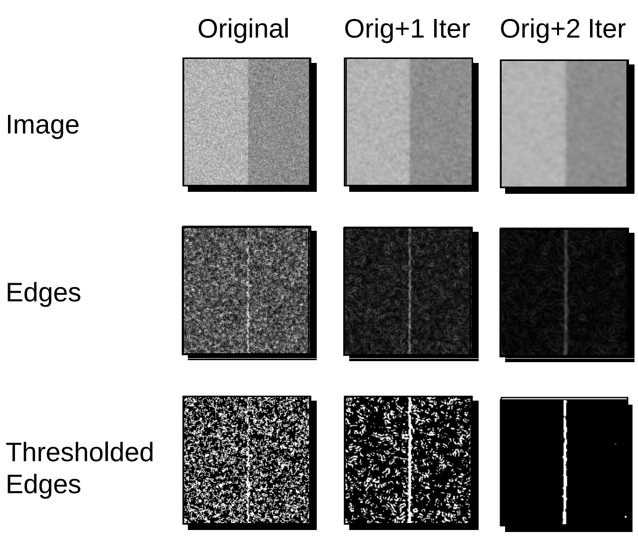
Smoothing and Edge Detection

- Derivatives are 'noisy' operations
  - edges are a high spatial frequency phenomenon
  - edge detectors are sensitive to and accent noise
- Averaging reduces noise
  - spatial averages can be computed using masks

$$\frac{1}{9}\times\begin{array}{|c|c|c|}\hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline\end{array} \qquad \frac{1}{8}\times\begin{array}{|c|c|c|}\hline 1 & 1 & 1 \\ \hline 1 & 0 & 1 \\ \hline 1 & 1 & 1 \\ \hline\end{array}$$

- Combine smoothing with edge detection.

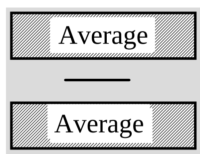
Effect of Blurring

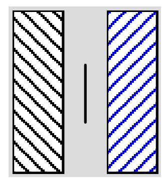


Combining the Two

- Applying this mask is equivalent to taking the difference of averages on either side of the central pixel.

-1	-1	-1
0	●	0
1	1	1





Many Different Kernels

- Variables
  - Size of kernel
  - Pattern of weights
- 1x2 Operator (we've already seen this one

$\Delta_j I = \begin{bmatrix} -1 & 1 \end{bmatrix}$ $\quad$ $\Delta_i I = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

Roberts Cross Operator

- Does not return any information about the orientation of the edge

$$S = \sqrt{[\,I(x, y) - I(x+1, y+1)\,]^2 + [\,I(x, y+1) - I(x+1, y)\,]^2}$$

or

$$S = |\,I(x, y) - I(x+1, y+1)\,| + |\,I(x, y+1) - I(x+1, y)\,|$$

$$\left| \begin{array}{|c|c|} \hline 1 & 0 \\ \hline 0 & -1 \\ \hline \end{array} \right| + \left| \begin{array}{|c|c|} \hline 0 & 1 \\ \hline -1 & 0 \\ \hline \end{array} \right|$$

Sobel Operator

$$S_1 = \begin{array}{|c|c|c|} \hline -1 & -2 & -1 \\ \hline 0 & 0 & 0 \\ \hline 1 & 2 & 1 \\ \hline \end{array} \qquad S_2 = \begin{array}{|c|c|c|} \hline -1 & 0 & 1 \\ \hline -2 & 0 & 2 \\ \hline -1 & 0 & 1 \\ \hline \end{array}$$

$$\text{Edge Magnitude} = \sqrt{S_1^2 + S_2^2}$$

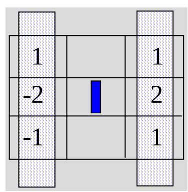
$$\text{Edge Direction} = \tan^{-1}\left(\frac{S_1}{S_2}\right)$$

Anatomy of the Sobel

$$\frac{1}{4}\begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} = 1/4 * [-1\ \ 0\ +1] \circledast \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

$$\frac{1}{4}\begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix} = 1/4 * [\ 1\ \ 2\ \ 1] \circledast \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

Sobel kernel
is separable!



Averaging done parallel to edge

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Prewitt Operator

$$P_1 = \begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} \qquad P_2 = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

$$\text{Edge Magnitude} = \sqrt{P_1^2 + P_2^2}$$

$$\text{Edge Direction} = \tan^{-1}\left(\frac{P_1}{P_2}\right)$$

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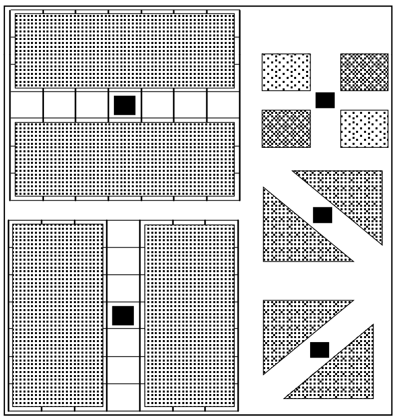
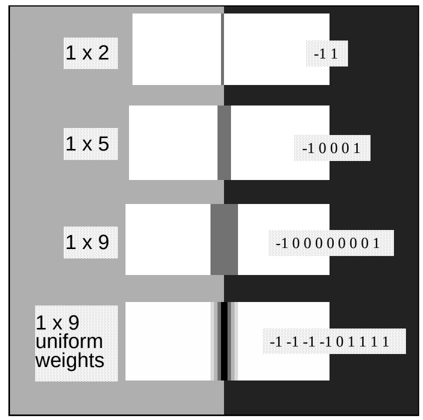
Large Masks

What happens as the mask size increases?





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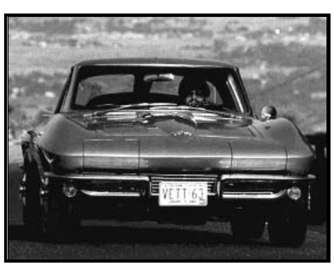
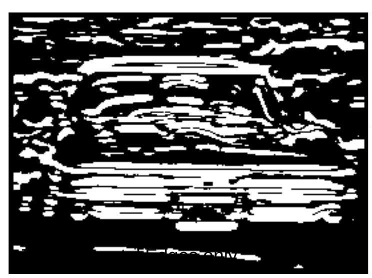
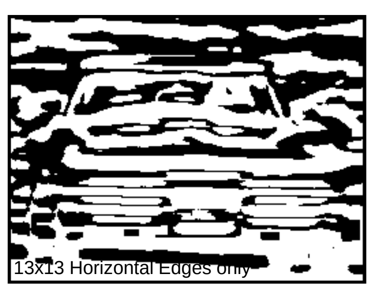
Large Kernels



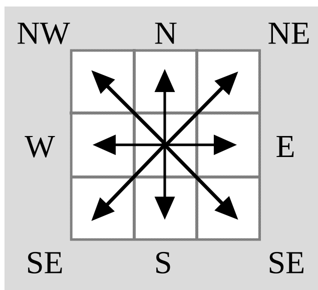


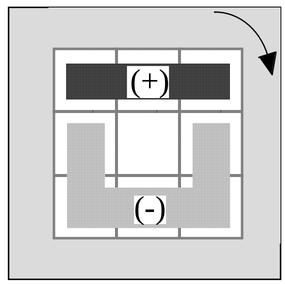


13X13 Horizontal Edges only

Compass Masks

- Use eight masks aligned with the usual compass directions
- Select largest response (magnitude)
- Orientation is the direction associated with the largest response





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Many Different Kernels

1	1	1
1	-2	1
-1	-1	-1

Prewitt 1

5	5	5
-3	0	-3
-3	-3	-3

Kirsch

-1	$-\sqrt{2}$	-1
0	0	0
1	$\sqrt{2}$	1

Frei & Chen

1	1	1
0	0	0
-1	-1	-1

Prewitt 2

1	2	1
0	0	0
-1	-2	-1

Sobel

Robinson Compass Masks

-1	0	1
-2	0	2
-1	0	1

→

0	1	2
-1	0	1
-2	-1	0

↗

1	2	1
0	0	0
-1	-2	-1

↑

2	1	0
1	0	-1
0	-1	-2

↖

1	0	-1
2	0	-2
1	0	-1

←

0	-1	-2
-1	0	-1
2	1	0

↙

-1	-2	-1
0	0	0
1	2	1

↓

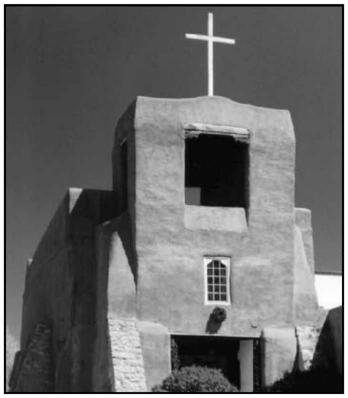
-2	-1	0
-1	0	1
0	1	2

↘

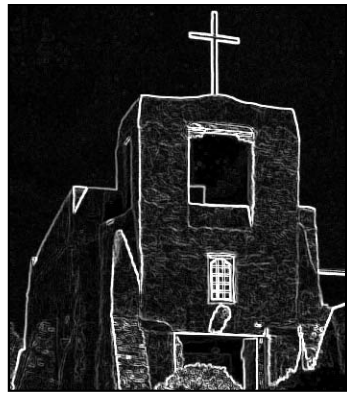
Analysis of Edge Kernels

- Analysis based on a step edge inclined at an angle θ (relative to y-axis) through center of window.
- Robinson/Sobel: true edge contrast less than 1.6% different from that computed by the operator.
- Error in edge direction
  - Robinson/Sobel: less than 1.5 degrees error
  - Prewitt: less than 7.5 degrees error
- Summary
  - Typically, 3 x 3 gradient operators perform better than 2 x 2.
  - Prewitt2 and Sobel perform better than any of the other 3x3 gradient estimation operators.
  - In low signal to noise ratio situations, gradient estimation operators of size larger than 3 x 3 have improved performance.
  - In large masks, weighting by distance from the central pixel is beneficial.

Prewitt Example



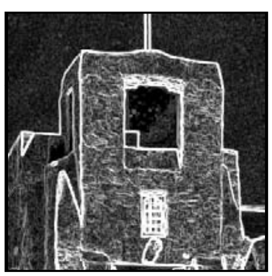
Santa Fe Mission

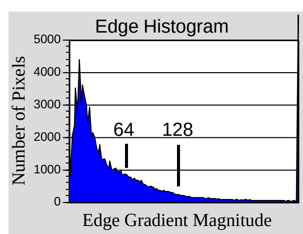


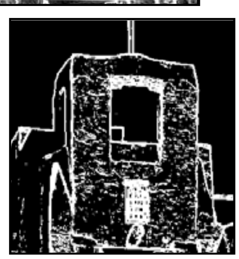
Prewitt Horizontal
and Vertical Edges
Combined

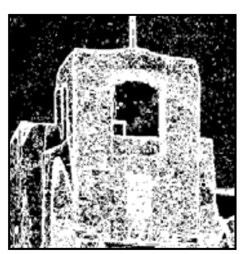
Edge Thresholding

- Global approach









See Haralick paper for thresholding based on statistical significance tests.

Demo in Photoshop

- Go through slides 40-71 after class
- Reading: Chapters 4 and 5
- Homework 2: Due after two weeks

You may try different operators in Photoshop, but do your homework by programming … …

Canny Edge Detector

- Probably most widely used
- LF. Canny, "A computational approach to edge detection", IEEE Trans. Pattern Anal. Machine Intelligence (PAMI), vol. PAMI vii-g, pp. 679-697, 1986.
- Based on a set of criteria that should be satisfied by an edge detector:
  - Good detection. There should be a minimum number of false negatives and false positives.
  - Good localization. The edge location must be reported as close as possible to the correct position.
  - Only one response to a single edge.

⇩

Cost function which could be optimized using variational methods

Canny Results

σ=1, T2=255, T1=1

```
I = imread('image file name');
BW1 = edge(I,'sobel');
BW2 = edge(I,'canny');
imshow(BW1)
figure, imshow(BW2)
```

'Y' or 'T' junction problem with Canny operator

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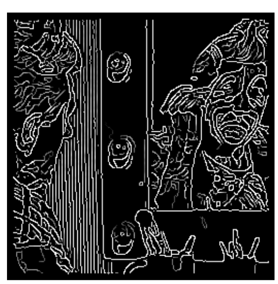
Canny Results

σ=1, T2=255, T1=220



σ=1, T2=128, T1=1

σ=2, T2=128, T1=1

M. Heath, S. Sarkar, T. Sanocki, and K.W. Bowyer, "A Robust Visual Method for Assessing the Relative Performance of Edge-Detection Algorithms" IEEE Transactions on Pattern Analysis and Machine Intelligence, Vol. 19, No. 12, December 1997, pp. 1338-1359.

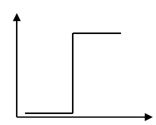
http://marathon.csee.usf.edu/edge/edge_detection.html

- Second derivatives…

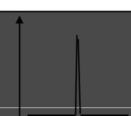
Edges from Second Derivatives

- Digital gradient operators estimate the first derivative of the image function in two or more directions.

f(x) = step edge



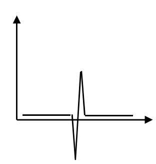
1st Derivative f'(x)



maximum

GRADIENT METHODS

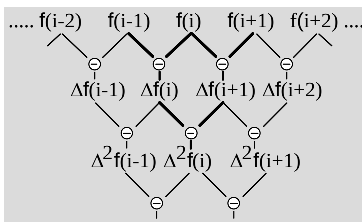
2nd Derivative f''(x)



zero crossing

Second Derivatives

- Second derivative = rate of change of first derivative.
- Maxima of first derivative = zero crossings of second derivative.
- For a discrete function, derivatives can be approximated by differencing.
- Consider the one dimensional case:



$\Delta^2 f(i) = \Delta\ f(i+1) - \Delta\ f(i)$

$= f(i+1) - 2\ f(i) + f(i-1)$

Mask:

1	**-2**	**1**

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Laplacian Operator

- Now consider a two-dimensional function f(x,y).
- The second partials of f(x,y) are not isotropic.
- Can be shown that the smallest possible isotropic second derivative operator is the Laplacian:

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

- Two-dimensional discrete approximation is:

	1	
1	-4	1
	1	

Example Laplacian Kernels

-1	-1	-1	-1	-1
-1	-1	-1	-1	-1
-1	-1	24	-1	-1
-1	-1	-1	-1	-1
-1	-1	-1	-1	-1

5X5

-1	-1	-1	-1	-1	-1	-1	-1	-1
-1	-1	-1	-1	-1	-1	-1	-1	-1
-1	-1	-1	-1	-1	-1	-1	-1	-1
-1	-1	-1	+8	+8	+8	-1	-1	-1
-1	-1	-1	+8	+8	+8	-1	-1	-1
-1	-1	-1	+8	+8	+8	-1	-1	-1
-1	-1	-1	-1	-1	-1	-1	-1	-1
-1	-1	-1	-1	-1	-1	-1	-1	-1
-1	-1	-1	-1	-1	-1	-1	-1	-1

9X9

- Note that these are not the optimal approximations to the Laplacian of the sizes shown.

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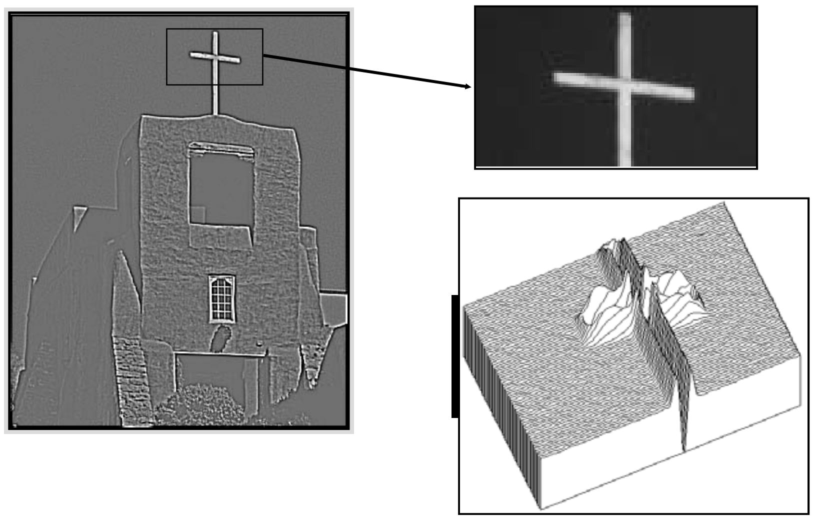
Example Application

5x5 Laplacian Filter

9x9 Laplacian Filter

Detailed View of Results



Interpretation of the Laplacian

- Consider the definition of the discrete Laplacian:

$$\nabla^2 I = \underbrace{I(i+1,j)+I(i-1,j)+I(i,j+1)+I(i,j-1)}_{\text{looks like a window sum}} - 4I(i,j)$$

- Rewrite as:

$$\nabla^2 I = I(i+1,j)+I(i-1,j)+I(i,j+1)+I(i,j-1)+I(i,j) - 5I(i,j)$$

- Factor out -5 to get:

$$\nabla^2 I = -5 \{I(i,j) - \text{window average}\}$$

- Laplacian can be obtained, up to the constant -5, by subtracting the average value around a point (i,j) from the image value at the point (i,j)!
  - What window and what averaging function?

Enhancement using the Laplacian

- The Laplacian can be used to enhance images:

$$I(i,j) - \nabla^2 I(i,j) = 5\,I(i,j) - [I(i+1,j) + I(i-1,j) + I(i,j+1) + I(i,j-1)]$$

- If (i,j) is in the middle of a flat region or long ramp: $I-\nabla^2 I = I$
- If (i,j) is at low end of ramp or edge: $I-\nabla^2 I < I$
- If (i,j) is at high end of ramp or edge: $I-\nabla^2 I > I$

- Effect is one of deblurring the image

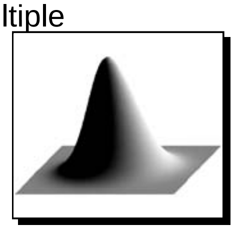
Laplacian Enhancement

Blurred Original

3x3 Laplacian Enhanced

Noise

- Second derivative, like first derivative, enhances noise
- Combine second derivative operator with a smoothing operator.
- Questions:
  - Nature of optimal smoothing filter.
  - How to detect intensity changes at a given scale.
  - How to combine information across multiple scales.
- Smoothing operator should be
  - 'tunable' in what it leaves behind
  - smooth and localized in image space.
- One operator which satisfies these two

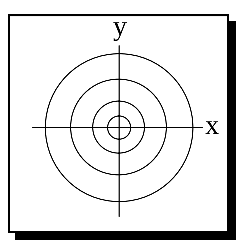


2D Gaussian Distribution

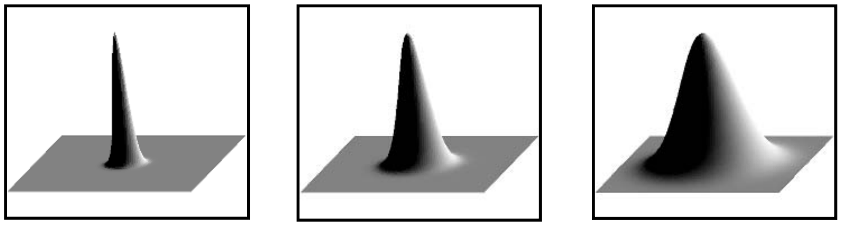
- The two-dimensional Gaussian distribution is defined by:

$$G(x,y) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\left[\frac{(x^2+y^2)}{2\sigma^2}\right]}$$

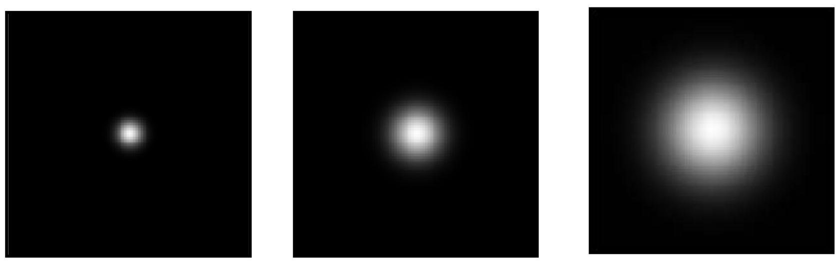
- From this distribution, can generate smoothing masks whose width depends upon σ:



σ Defines Kernel 'Width'



$\sigma^2 = .25$ $\sigma^2 = 1.0$ $\sigma^2 = 4.0$



Creating Gaussian Kernels

- The mask weights are evaluated from the Gaussian distribution:

$$W(i,j) = k * \exp\left(- \frac{i^2 + j^2}{2\,\sigma^2}\right)$$

- This can be rewritten as:

$$\frac{W(i,j)}{k} = \exp\left(- \frac{i^2 + j^2}{2\,\sigma^2}\right)$$

- This can now be evaluated over a window of size nxn to obtain a kernel in which the (0,0) value is 1.
- k is a scaling constant

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Example

- Choose σ^2 = 2. and n = 7, then:

i \ j	-3	-2	-1	0	1	2	3
-3	.011	.039	.082	.105	.082	.039	.011
-2	.039	.135	.287	.368	.287	.135	.039
-1	.082	.287	.606	.779	.606	.287	.082
0	.105	.039	.779	1.000	.779	.368	.105
1	.082	.287	.606	.779	.606	$\boxed{.287}$	.082
2	.039	.135	.287	.368	.287	.135	.039
3	.011	.039	.082	.105	.082	.039	$\boxed{.011}$

$$\frac{W(1,2)}{k} = \exp\left(-\frac{1^2 + 2^2}{2*2}\right)$$

To make this value 1, choose k = 91.

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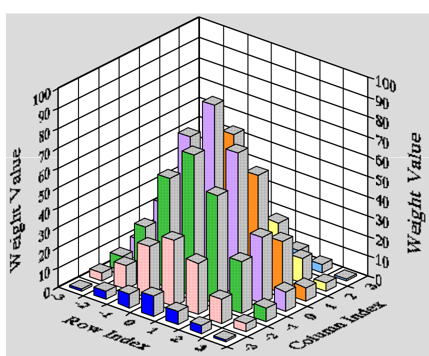
Example

1	4	7	10	7	4	1
4	12	26	33	26	12	4
7	26	55	71	55	26	7
10	33	71	91	71	33	10
7	26	55	71	55	26	7
4	12	26	33	26	12	4
1	4	7	10	7	4	1

7x7 Gaussian Filter

$$\sum_{i=-3}^{3}\sum_{j=-3}^{3} W(i,j) = 1,115$$

Plot of Weight Values



Kernel Application

7x7 Gaussian Kernel

15x15 Gaussian Kernel

Why Gaussian for Smoothing

- Gaussian is not the only choice, but it has a number of important properties
  - If we convolve a Gaussian with another Gaussian, the result is a Gaussian
    - This is called linear scale space

$$G_{\sigma_1} * G_{\sigma_2} = G_{\sqrt{\sigma_1^2+\sigma_2^2}}.$$

  - Efficiency: separable
  - Central limit theorem

Why Gaussian for Smoothing

- Gaussian is separable

$$G_\sigma(x,y) = \frac{1}{2\pi\sigma^2}\exp\left(-\frac{(x^2+y^2)}{2\sigma^2}\right)$$
$$= \left(\frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{(x^2)}{2\sigma^2}\right)\right) \times \left(\frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{(y^2)}{2\sigma^2}\right)\right),$$

Why Gaussian for Smoothing – cont.

- Gaussian is the solution to the diffusion equation

$$\frac{\partial \Phi}{\partial \sigma} = \frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} = \nabla^2 \Phi,$$
$$\Phi(x, y, 0) = \mathcal{I}(x, y)$$

- We can extend it to non-linear smoothing

$$\frac{\partial \Phi}{\partial \sigma} = \nabla \cdot (c(x, y, \sigma)\nabla\Phi)$$
$$= c(x, y, \sigma)\nabla^2\Phi + (\nabla c(x, y, \sigma)) \cdot (\nabla\Phi)$$

∇^2G Filter

- Marr and Hildreth approach:
  1. Apply Gaussian smoothing using σ's of increasing size:

     $$G \circledast I$$

  2. Take the Laplacian of the resulting images:

     $$\nabla^2 (G \circledast I)$$

  3. Look for zero crossings.

- Second expression can be written as: $(\nabla^2 G) \circledast I$

- Thus, can take Laplacian of the Gaussian and use that as the operator.

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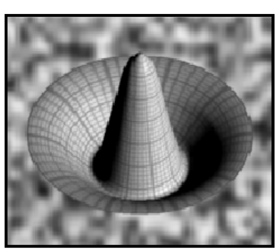
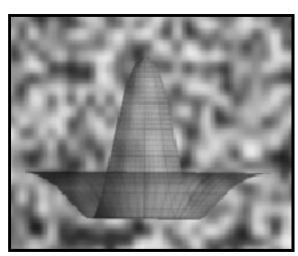
Mexican Hat Filter

- Laplacian of the Gaussian

$$\nabla^2 G(x,y) = \frac{-1}{\pi\sigma^4}\left[1 - \frac{(x^2+y^2)}{2\sigma^2}\right] e^{-\left[\frac{(x^2+y^2)}{2\sigma^2}\right]}$$

- ∇^2G is a circularly symmetric operator.
- Also called the hat or Mexican-hat operator.





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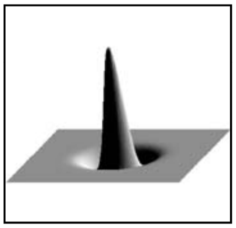
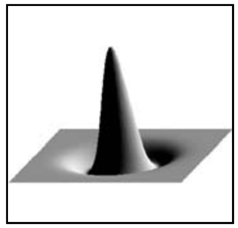
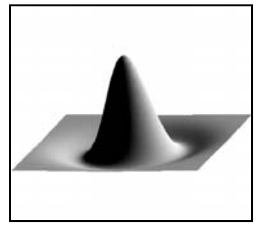
σ^2 Controls Size



$\sigma^2 = 0.5$



$\sigma^2 = 1.0$



$\sigma^2 = 2.0$

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Kernels

5x5

0	0	-1	0	0
0	-1	-2	-1	0
-1	-2	16	-2	-1
0	-1	-2	-1	0
0	0	-1	0	0

17 x 17

0	0	0	0	0	0	-1	-1	-1	-1	-1	0	0	0	0	0	0
0	0	0	0	-1	-1	-1	-1	-1	-1	-1	-1	-1	0	0	0	0
0	0	-1	-1	-1	-2	-3	-3	-3	-3	-3	-2	-1	-1	-1	0	0
0	0	-1	-1	-2	-3	-3	-3	-3	-3	-3	-3	-2	-1	-1	0	0
0	-1	-1	-2	-3	-3	-3	-2	-3	-2	-3	-3	-3	-2	-1	-1	0
0	-1	-2	-3	-3	-3	0	2	4	2	0	-3	-3	-3	-2	-1	0
-1	-1	-3	-3	-3	0	4	10	12	10	4	0	-3	-3	-3	-1	-1
-1	-1	-3	-3	-2	2	10	18	21	18	10	2	-2	-3	-3	-1	-1
-1	-1	-3	-3	-3	4	12	21	24	21	112	4	-3	-3	-3	-1	-1
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0	-1	-2	-3	-3	-3	0	2	4	2	0	-3	-3	-3	-2	-1	0
0	-1	-1	-2	-3	-3	-3	-3	-3	-3	-3	-3	-3	-2	-1	-1	0
0	0	-1	-1	-2	-3	-3	-3	-3	-3	-3	-3	-2	-1	-1	0	0
0	0	-1	-1	-1	-2	-3	-3	-3	-3	-3	-2	-1	-1	-1	0	0
0	0	0	0	-1	-1	-1	-1	-1	-1	-1	-1	-1	0	0	0	0

- Remember the center surround cells in the human system?

Example

13x13 Kernel

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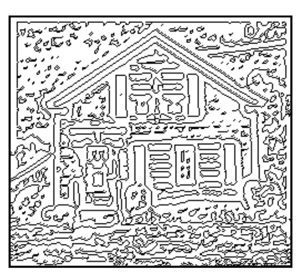
Example

13 x 13 Hat Filter

Thesholded Positive

Thesholded Negative



Zero Crossings

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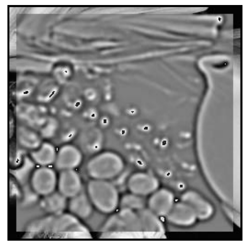
Scale Space



17x17 LoG Filter

Thresholded Positive

Thresholded Negative

Zero Crossings

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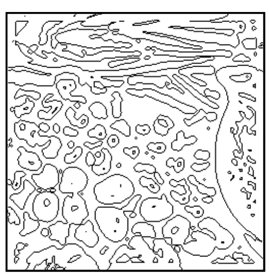
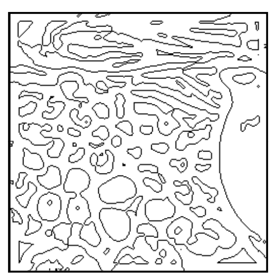
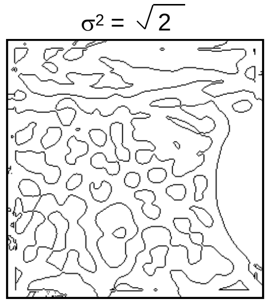
Scale Space



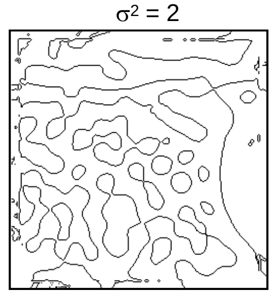
$\sigma^2 = \sqrt{2}$



$\sigma^2 = 2$



$\sigma^2 = 2\sqrt{2}$



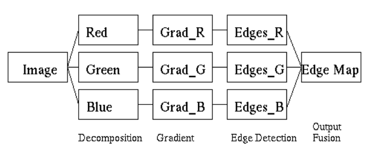
$\sigma^2 = 4$

Multi-Resolution Scale Space

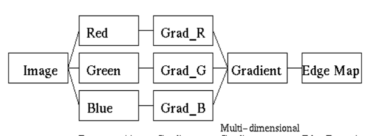
- Observations:
  - For sufficiently different σ 's, the zero crossings will be unrelated unless there is 'something going on' in the image.
  - If there are coincident zero crossings in two or more successive zero crossing images, then there is sufficient evidence for an edge in the image.
  - If the coincident zero crossings disappear as σ becomes larger, then either:
    - two or more local intensity changes are being averaged together, or
    - two independent phenomena are operating to produce intensity changes in the same region of the image but at different scales.
- Use these ideas to produce a 'first-pass' approach to edge detection using multi-resolution zero crossing data.
- Never completely worked out
- See Tony Lindbergh's thesis and papers

Color Edge Detection

- Typical Approaches
  - Fusion of results on R, G, B separately



  - Multi-dimensional gradient methods



  - Vector methods
  - Color signatures: Stanford (Rubner and Thomasi)

Hierarchical Feature Extraction

- Most features are extracted by combining a small set of primitive features (edges, corners, regions)
  - Grouping: which edges/corners/curves form a group?
    - perceptual organization at the intermediate-level of vision
  - Model Fitting: what structure best describes the group?
- Consider a slightly simpler problem…..

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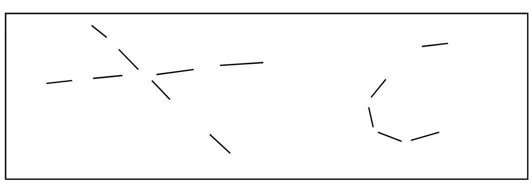
From Edgels to Lines

- Given local edge elements:



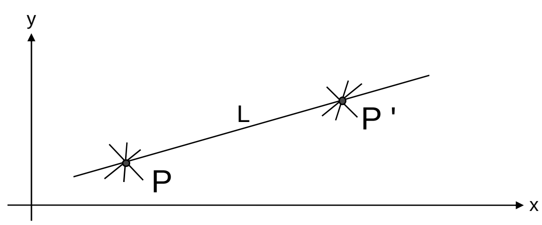
- Can we organize these into more 'complete' structures, such as straight lines?
- Group edge points into lines?
- Consider a fairly simple technique...

Edgels to Lines

- Given a set of local edge elements
  - With or without orientation information
- How can we extract longer straight lines?
- General idea:
  - Find an alternative space in which lines map to points
  - Each edge element 'votes' for the straight line which it may be a part of.
  - Points receiving a high number of votes might correspond to actual straight lines in the image.
- The idea behind the Hough transform is that a change in representation converts a point grouping problem into a peak detection problem

Edgels to Lines

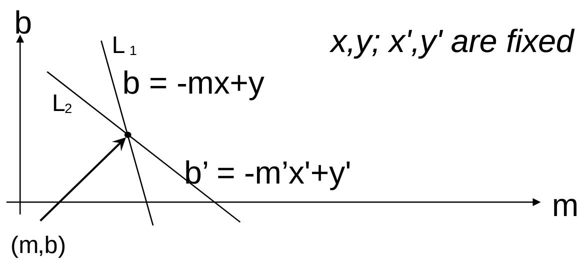
- Consider two (edge) points, P(x,y) and P'(x',y') in image space:



- The set of all lines through P=(x,y) is y=mx + b, for appropriate choices of m and b.
  - Similarly for P'
- But this is also the equation of a line in (m,b) space, or parameter space.

Parameter Space

- The intersection represents the parameters of the equation of a line y=mx+b going through both (x,y) and (x',y').



- The more colinear edgels there are in the image, the more lines will intersect in parameter space
- Leads directly to an algorithm

General Idea

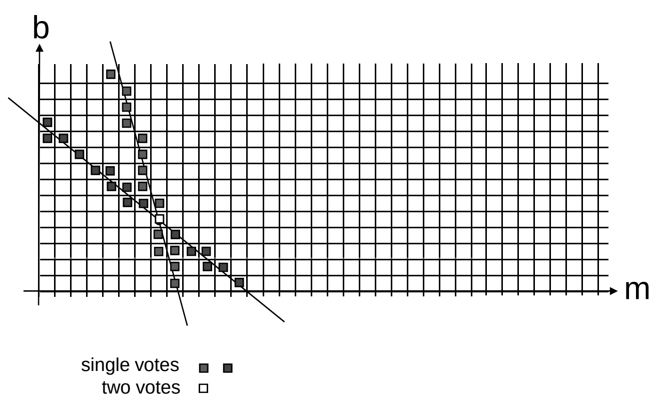
- General Idea:
  - The Hough space (m,b) is a representation of every possible line segment in the plane
  - Make the Hough space (m and b) discrete
  - Let every edge point in the image plane 'vote for' any line it might belong to.

Hough Transform

- Line Detection Algorithm: Hough Transform
  - Quantize b and m into appropriate 'buckets'.
    - Need to decide what's 'appropriate'
  - Create accumulator array H(m,b), all of whose elements are initially zero.
  - For each point (i,j) in the edge image for which the edge magnitude is above a specific threshold, increment all points in H(m,b) for all discrete values of m and b satisfying b = -mj+i.
    - Note that H is a two dimensional histogram
  - Local maxima in H corresponds to colinear edge points in the edge image.

Quantized Parameter Space

- Quantization



The problem of line detection in image space has been transformed into the problem of cluster detection in parameter space

Example

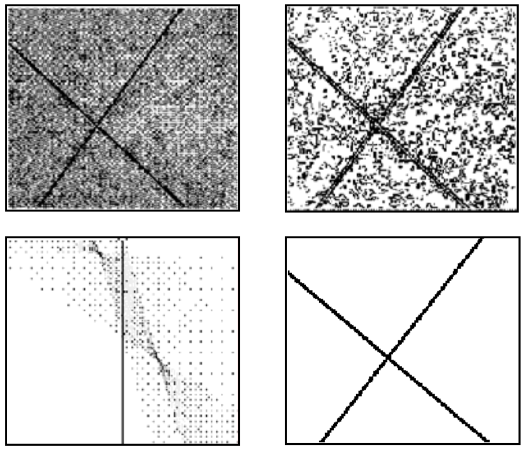
- The problem of line detection in image space has been transformed into the problem of cluster detection in parameter space

Image

Edges

Accumulator Array

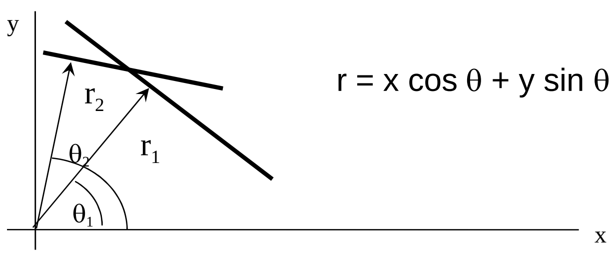
Result



Problems

- Vertical lines have infinite slopes
  - difficult to quantize m to take this into account.
- Use alternative parameterization of a line
  - polar coordinate representation

$$r_1 = x_1 \cos(\theta) + y_1 \sin(\theta)$$



$r = x \cos \theta + y \sin \theta$

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Why?

- (ρ,θ) is an efficient representation:
  - Small: only two parameters (like y=mx+b)
  - Finite: $0 \le \rho \le \sqrt{(row^2+col^2)}$, $0 \le \theta \le 2\pi$
  - Unique: only one representation per line

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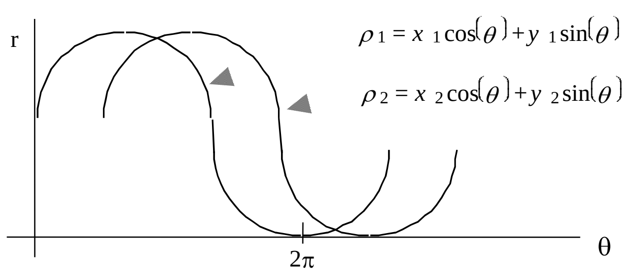
Alternate Representation

- Curve in (ρ,θ) space is now a sinusoid
  - but the algorithm remains valid.

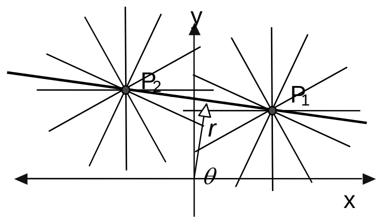


Example

$r = -3\cos(\theta) + 5\sin(\theta)$

$r = 4\cos(\theta) + 4\sin(\theta)$

Two Constraints



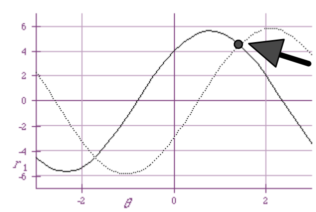
$P_1 = (4, 4)$

$P_2 = (-3, 5)$

$r = 4c + 4s$	$s = \frac{7}{50}\sqrt{50}$	$\theta = 1.4289$
$r = -3c + 5s$	$c = \frac{1}{50}\sqrt{50}$	$r = 4.5255$
$s^2 + c^2 = 1$		

Solve for r and θ

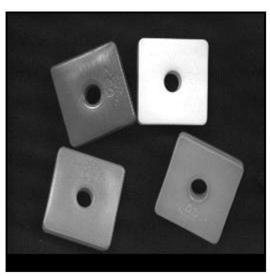
(r, θ) Space

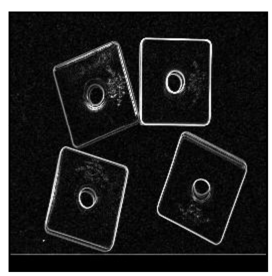


(r, θ)

Real Example

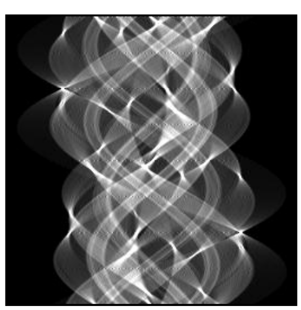
Image

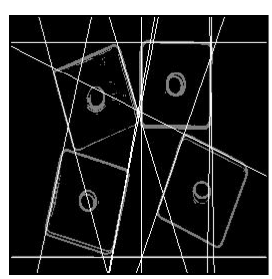




Edges

Accumulator Array

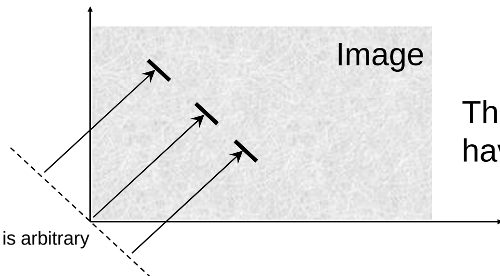




Result

Modifications

- Note that this technique only uses the fact that an edge exists at point (i,j).
- What about the orientation of the edge?
  - More constraints!

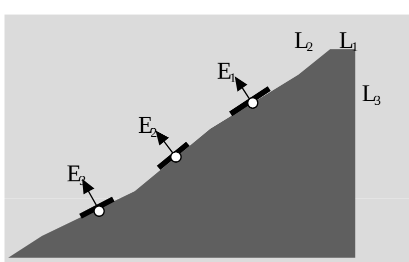


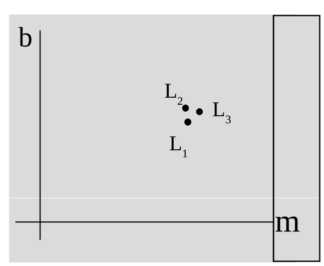
The three edges have same (r, θ)

- Use estimate of edge orientation as θ.
- Each edgel now maps to a point in Hough space.

Gradient Data

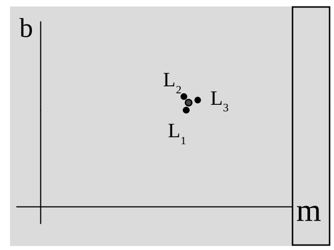
- Colinear edges in Cartesian coordinate space now form point clusters in (m,b) parameter space.



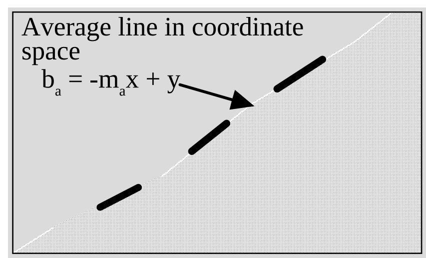


Gradient Data

- ‘Average’ point in Hough Space:

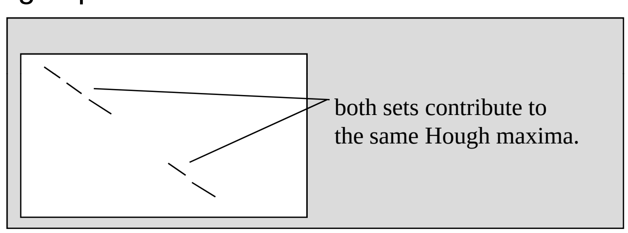


- Leads to an ‘average’ line in image space:



Post Hough

- Image space localization is lost:



- Consequently, we still need to do some image space manipulations, e.g., something like an edge 'connected components' algorithm.
- Heikki Kälviäinen, Petri Hirvonen, L. Xu and Erkki Oja, “Probabilistic and nonprobabilistic Hough Transforms: Overview and comparisons”, *Image and vision computing*, Volume 13, Number 4, pp. 239-252, May 1995.

Hough Fitting

- Sort the edges in one Hough cluster
  - rotate edge points according to θ
  - sort them by (rotated) x coordinate
- Look for Gaps
  - have the user provide a “max gap” threshold
  - if two edges (in the sorted list) are more than max gap apart, break the line into segments
  - if there are enough edges in a given segment, fit a straight line to the points

Generalizations

- Hough technique generalizes to any parameterized curve:

$$f(x,\underbrace{a}_{\text{parameter vector (axes in Hough space)}}) = 0$$

- Success of technique depends upon the quantization of the parameters:
  - too coarse: maxima 'pushed' together
  - too fine: peaks less defined
- Note that exponential growth in the dimensions of the accumulator array with the the number of curve parameters restricts its practical application to curves with few parameters

Example: Finding a Circle

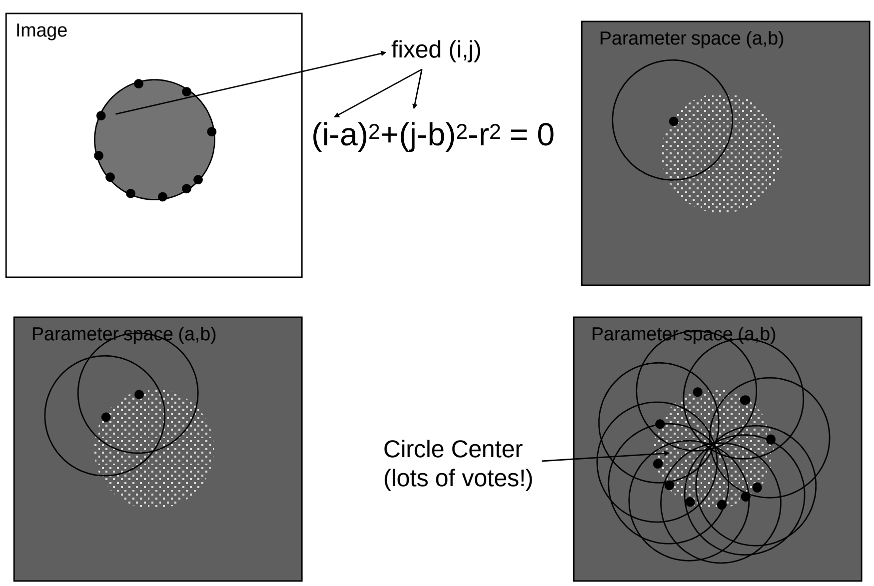
- Circles have three parameters
  - Center (a,b)
  - Radius r
- Circle $f(x,y,r) = (x-a)^2+(y-b)^2-r^2 = 0$
- Task:

> Find the center of a circle with known radius r given an edge image with no gradient direction information (edge location only)

- Given an edge point at (x,y) in the image, where could the center of the circle be?

3D Computer Vision
and Video Computing

Finding a Circle



Finding Circles

- If we don't know r, accumulator array is 3-dimensional
- If edge directions are known, computational complexity if reduced
  - Suppose there is a known error limit on the edge direction (say +/- $10^{o)}$ - how does this affect the search?
- Hough can be extended in many ways….see, for example:
  - Ballard, D. H. Generalizing the Hough Transform to Detect Arbitrary Shapes, Pattern Recognition 13:111-122, 1981.
  - Illingworth, J. and J. Kittler, Survey of the Hough Transform, Computer Vision, Graphics, and Image Processing, 44(1):87-116, 1988